Investigating Recurrent Neural Network Memory Structures using Neuro-Evolution

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Abstract—This paper presents a new algorithm, Evolutionary eXploration of Augmenting Memory Models (EXAMM), which is capable of evolving recurrent neural networks (RNNs) using a wide variety of memory structures, such as Δ -RNN, GRU, LSTM, MGU and UGRNN cells. EXAMM evolved RNNs to perform prediction of large-scale, real world time series data from the aviation and power industries. These data sets consist of very long time series (thousands of readings), each with a large number of potentially correlated and dependent parameters. Four different parameters were selected for prediction and EXAMM runs were performed using each memory cell type alone, each cell type and simple neurons, and with all possible memory cell types and simple neurons. Evolved RNN performance was measured using repeated k-fold cross validation, resulting in 2420 EXAMM runs which evolved 4,840,000 RNNs in $\sim 24,200$ CPU hours on a high performance computing cluster. Generalization of the evolved RNNs was examined statistically, providing interesting findings that can help refine the RNN memory cell design as well as inform future neuro-evolution algorithms development.

I. INTRODUCTION

This work is motivated by a major open question in the field of artificial neural network (ANN) research: *What neural memory structures appear to be optimal for time-series prediction?* Conducting such a search over ANN structures entails manual, primarily human-driven labor and activity. As more advances are made in the field, the number of possible architecture variations and modifications explodes combinatorially. The growing space of architecture structures combined with the limited, often simple heuristic local search that can be conducted by human experts means that efficiently finding architectures that generalize well while still maintaining low parameter complexity (given that regularization is important for most data sample sizes) makes for a nearly impossible, largely intractable search problem.

In natural biological systems, the process of evolution, over long time-spans, endows organisms with various inductive biases that allow them to adapt and learn their environment quickly and readily. It is thought that these inductive biases are what provide infants the ability to quickly learn complex pattern recognition/detection functions with limited data across various sensory modalities [1], [2], such as in visual and speech sensory domains. While artificial forms of evolution, such as the classical genetic algorithm [3], are significantly simplified from the actual evolutionary process that creates useful inductive biases to drive development and survival of organisms at large, these optimization procedures offer the chance to develop non-human centered ways of generating useful and even potentially optimal neural architectures.

While neuro-evolution (applying evolutionary processes to the development of ANNs) been used in searching the space of feed forward and even convolutional architectures for tasks involving static inputs [4]–[13], less effort has been put into exploring the evolution of recurrent memory structures that operate with complex time based data sequences and uncovering what forms and structures the neuro-evolutionary process finds. Insights extracted from the evolved architectures can serve as inductive biases for subsequent research in neural network design and development.

The evolution of recurrent neural networks (RNNs) poses significant challenges above beyond the already challenging task of evolving feed forward and convolutional neural networks. First, RNNs are more challenging to train due to issues with exploding and vanishing gradients which occur when unrolling a RNN over a long time series with the backpropagation through time (BPTT) algorithm [14]. Due to this issue, development of recurrent memory cells which can preserve memory and long term dependencies while alleviating the exploding and vanishing gradient problem has been an area of significant study [15]-[19]. Further, as in the case of timevarying data, input samples are strictly ordered in time and thus induce long-term dependencies that any useful stateful adaptive process/model must extract in order to generalize. The complexity of time-series tasks varies, entailing the prediction of a particular variable of interest over time or even constructing a full generative model of all the available variables, perhaps additionally involving other non-trivial tasks such as data imputation.

Due to these issues, RNNs for time series data prediction can require the use of a variety of memory cell structures in addition to recurrent connections spanning different time spans. To optimize within this large search space, Evolutionary eXploration of Augmenting Memory Models (EXAMM) integrates an extensible collection of different complex memory cell types with simple neuronal building blocks and recurrent connections of varying time spans. EXAMM was used to evolve RNNs with various cell types to predict time series data from two real-world, large-scale data sets.

In total, 2420 EXAMM runs were done using repeated kfold cross valdation, generating a large set of evolved RNNs to analyze statistically. Results are particularly interesting in that while they show allowing selection from any of the available memory cell structures can provide very well performing networks in many test cases; it does come at the cost of reliability (how well the evolved RNNs perform in an average case). Further, even small modifications to the neuro-evolutionary process, such as allowing simple neurons, can generally improve predictions, but they can also have unintended consequences - in some examples it meant the difference between a memory cell structure performing the best as opposed to the worst. The authors hope that these results can help guide not only the further development of neuro-evolution algorithms, but also help refine and inform the development of new memory cell structures and human designed RNNs.

II. EVOLVING RECURRENT NETWORKS

Several methods for evolving NN topologies along with weights have been searched and deployed, with NeuroEvolution of Augmenting Topologies (NEAT) [12] perhaps being the most well-known. EXAMM differs from NEAT in that it includes more advanced node-level mutations (see Section III-A2), and utilizes Lamarckian weight initialization (see Section III-B) along with backpropagation to evolve the weights as opposed to a simpler less efficient genetic strategy. Additionally, it has been developed with large-scale concurrency in mind, and utilizes an asynchronous steady state approach which has been shown to facilitate scalability to potentially millions of compute nodes [20].

Other recent work by Rawal and Miikkulainen has investigated an information maximization objective [21] strategy to evolve RNNs. This strategy essentially utilizes NEAT with Long Short Term Memory (LSTM) neurons instead of regular neurons. EXAMM provides a more in-depth study of the performance of possible recurrent cell types as it examines both simple neurons and four other cell structures beyond LSTMs. Rawal and Miikulainen have also utilized tree-based encoding [22] to evolve recurrent cellular structures within fixed architectures built of layers of the evolved cell types. Combining this evolution of cell structure along with the overall RNN structure stands as interesting future work.

Ant colony optimization (ACO) has also been investigated as a way to select which connections should be utilized in RNNs and LSTM RNNs by Desell and ElSaid [23]–[25]. In particular, this ACO approach was shown to reduce the number of trainable connections in half while providing a significant improvement in predictions of engine vibration [24]. However, this approach works within a fixed RNN architecture and cannot evolve an overall RNN structure.

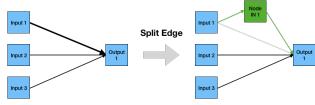
III. EVOLUTIONARY EXPLORATION OF AUGMENTING MEMORY MODELS

The EXAMM algorithm presented in this work expands on an earlier algorithm, EXALT [26] which can evolve RNNs with either simple neurons or LSTM cells. It further refines EXALT's mutation operations to reduce hyperparameters using statistical information from parental RNN genomes. Also, EXALT only used a single steady state population, and EXAMM expands on this to use islands, which have been shown by Alba and Tomassini to greatly improve performance of distributed evolutionary algorithms, potentially providing superlinear speedup [27]. A master process maintains each population of islands, and generates new RNNs form islands in a round robin manner to be trained upon request by workers. When a worker completes training a RNN, it is inserted into the island it was generated from if its fitness (mean squared error on the test data) is better than the worst in the island, and then the worst RNN in the island is removed. This asynchrony is particularly important as the generated RNNs will have different architectures, each taking a different amount of time to train. By having a master process control the population, workers can complete the training of the generated RNNs at whatever speed they can and the algorithm is naturally load balanced. This allows EXAMM to scale to the number of available processors, having a population size independent of processor availability, unlike synchronous parallel evolutionary strategies. The EXAMM codebase has a multithreaded implementation for multicore CPUs as well as an MPI [28] implementation for use on high performance computing resources.

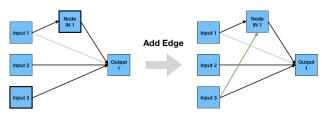
A. Mutation and Recombination Operations

RNNs are evolved with edge-level operations, as done in NEAT, as well as with new high level node mutations as in EXALT and EXACT. Whereas NEAT only requires innovation numbers for new edges, EXAMM requires innovation numbers for both new nodes, new edges and new recurrent edges. The master process keeps track of all node, edge and recurrent edge innovations made, which are required to perform the crossover operation in linear time without a graph matching algorithm. Figures 1 and 2 display a visual walkthrough of all the mutation operations used by EXAMM. Nodes and edges selected to be modified are highlighted, and then new elements to the RNN are shown in green. Edge innovation numbers are not shown for clarity. Enabled edges are in black, disabled edges are in grey.

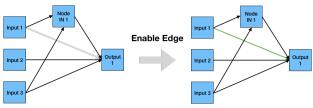
It should be noted that for the following operations, whenever an edge is added, unless otherwise specified, it is probabilistically selected to be a recurrent connection with the following recurrent probability: $p = \frac{n_{re}}{n_{ff}+n_{re}}$, where n_{re} is the number of enabled recurrent edges and n_{ff} is the number of enabled feed forward edges in the parent RNN. A recurrent connection will go back a randomly selected number of time steps with bound specified as search parameters (in this work, 1 to 10 time steps), allowing for recurrent connections of varying time spans. Any newly created node is selected



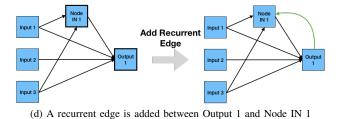
(a) The edge between Input 1 and Output 1 is selected to be split. A new node with innovation number (IN) 1 is created.

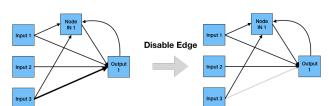


(b) Input 3 and Node IN 1 are selected to have an edge between them added.



(c) The edge between Input 3 and Output 1 is enabled.





(e) The edge between Input 3 and Output 1 is disabled.

Fig. 1: Edge mutation operations.

uniformly at random as a simple neuron or from the memory cell types specified by the EXAMM run input parameters.

1) Edge Mutations::

a) Disable Edge: This operation randomly selects an enabled edge or recurrent edge in a RNN genome and disables it so that it is not used. The edge remains in the genome. As the *disable edge* operation can potentially make an output node unreachable, after all mutation operations have been performed to generate a child RNN genome, if any output node is unreachable that RNN genome is discarded without training.

b) Enable Edge: If there are any disabled edges or recurrent edges in the RNN genome, this operation selects a disabled edge or recurrent edge at random and enables it.

c) Split Edge: This operation selects an enabled edge at random and disables it. It creates a new node and two new edges, and connects the input node of the split edge to the new node, and the new node to the output node of the split edge. If the split edge was recurrent, the new edges will also be recurrent (with the same time skip); otherwise they will be feed forward.

d) Add Edge: This operation selects two nodes n_1 and n_2 within the RNN Genome at random, such that $depth_{n_1} < depth_{n_2}$ and such that there is not already an edge between those nodes in this RNN Genome. Then it adds an edge from n_1 to n_2 .

e) Add Recurrent Edge: This operation selects two nodes n_1 and n_2 within the RNN Genome at random and then adds a recurrent edge from n_1 to n_2 , selecting a time span as described before. The same two nodes can be connected with multiple recurrent connections, each spanning different times; however it will not create a duplicate recurrent connection with the same time span.

2) Node Mutations::

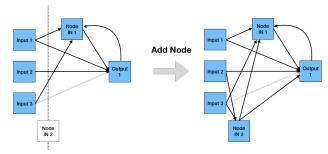
a) Disable Node: This operation selects a random nonoutput node and disables it along with all of its incoming and outgoing edges. Note that this allows for input nodes to be dropped out, which can be useful when it is not previously known which input parameters are correlated to the output.

b) Enable Node: This operation selects a random disabled node and enables it along with all of its incoming and outgoing edges.

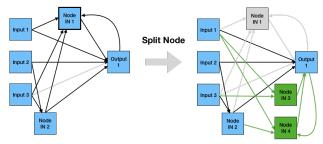
c) Add Node: This operation selects a random depth between 0 and 1, noninclusive. Given that the input node is always depth 0 and the output nodes are always depth 1, this depth will split the RNN in two. A new node is created at that depth, and the number of input and output edges and recurrent edges are generated using normal distributions with mean and variances equal to the mean and variances for the of input/output edges and recurrent edges of all nodes in the parent RNN.

d) Split Node: This operation takes one non-input, nonoutput node at random and splits it. This node is disabled (as in the disable node operation) and two new nodes are created at the same depth as their parent. At least one input and one output edge are assigned to each of the new nodes, of a duplicate type from the parent, with the others being assigned randomly, ensuring that the newly created nodes have both inputs and outputs.

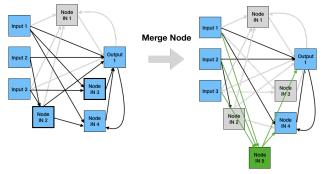
e) Merge Node: This operation takes two non-input, nonoutput nodes at random and combines them. Selected nodes are disabled (as in the disable node operation) and a new node is created at depth equal to the average of its parents. This node is connected to the inputs and outputs of its parents with a duplicate type from the parent; as input edges connected to lower depth nodes and output edges connect to greater depth nodes.



(a) A node with IN 2 is selected to be added at a depth between the inputs & Node IN 1. Edges are randomly added to Input 2 and 3, and Node IN 1 and Output 1.



(b) Node IN 1 is selected to be split. It is disabled with its input/output edges. It is split into Nodes IN 3 and 4, which get half the inputs. Both have an output edge to Output 1 since there was only one output from Node IN 1.



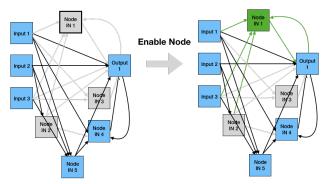
(c) Node IN 2 and 3 are selected for a merger (input/output edges are disabled). Node IN 5 is created with edges between all their inputs/outputs.

Fig. 2: Node mutation operations.

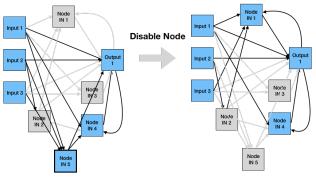
3) Other Operations::

a) Crossover: creates a child RNN using all reachable nodes and edges from two parents. A node or edge is reachable if there is a path of enabled nodes and edges from an input node to it as well as a path of enabled nodes and edges from it to an output node, *i.e.*, a node or edge is reachable if it actually affects the RNN. Crossover can be done either within an island (*intra-island*) or between islands (*inter-island*). Interisland crossover selects a random parent in the target island, and the best RNN from the other islands.

b) Clone: creates a copy of the parent genome, initialized to the same weights. This allows a particular genome to continue training in cases where further training may be more beneficial than performing a mutation or crossover.



(d) Node IN 1 is selected to be enabled, along with all its input and output edges.



(e) Node IN 5 is selected to be disabled, along with all its input and output edges.

Fig. 2: Node mutation operations (continued).

B. Lamarckian Weight Initialization

For RNNs generated during population initialization, the weights are initialized uniformly at random between -0.5 and 0.5. Biases and weights for new nodes and edges are initialized from a normal distribution based on the average, mu, and variance, σ^2 , of the parents' weights. However, RNNs generated through mutation or crossover re-use parental weights, allowing the RNNs to train from where the parents are left off, *i.e.*, "Lamarckian" weight initialization.

During crossover, in the case where an edge or node exists in both parents, the child weights are generated by recombining the parents' weights. Given a random number $-0.5 \le r \le 1.5$, a child's weight w_c is set to $w_c = r(w_{p2} - w_{p1}) + w_{p1}$, where w_{p1} is the weight from the more fit parent, and w_{p2} is the weight from the less fit parent. This allows the child weights to be set along a gradient calculated from the weights of the two parents.

This weight initialization strategy is particularly important as newly generated RNNs do not need to be completely retrained from scratch. In fact, the RNNs only need to be trained for a few epochs to investigate the benefits of newly added structures. In this work, the generated RNNs are only trained for 10 epochs (see Section V-B), where training a static RNN structure from scratch may require hundreds or even thousands of epochs.

C. A Collection of Memory Cells

Node types can range quite a bit in terms of complexity and their design largely governs the form of the underlying memory structure. Simple neurons can be evolved into generalized versions of traditional Elman and Jordan neurons as EXAMM adds recurrent connections. Below describes how simple neurons can evolve to generalized Elman and Jordan neurons, as well as complex cells such as the Delta-RNN, the minimally gated unit (MGU) [18], the update-gated RNN (UGRNN) [17], the Gated Recurrent Unit (GRU) [16], and Long Short Term Memory (LSTM) [15].

Elman, Jordan and Arbitrary Recurrent Connections: In EXAMM, simple neurons are represented as potential recurrent neurons with both recurrent and feed forward inputs. I is the set of all nodes with a feed forward connection to simple neuron j while R is the set of all nodes with a recurrent connection to simple neuron j. At time step t, the output is a weighted summation of all feed forward inputs, where w_{ij} is the feed forward weight connecting node i to node j, plus a weighted summation of recurrent inputs, where v_{rjk} is the recurrent weight from node r at time step t - k to the node j, where k is the time span of the recurrent connection. Thus the state function s for a computing a simple neuron is:¹

$$s_j(t) = \phi_s \bigg(\sum_{i \in I} w_{ij} \cdot s_i(t) + \sum_{r \in R,k} v_{rjk} \cdot s_r(t-k) \bigg)$$

The overall state is a linear combination of the projected input and an affine transformation of the vector summary of the past. The post-activation function, $\phi_s(\cdot)$, can be any differentiable element-wise function, however for the purposes of this work it was limited to the hyperbolic tangent $\phi(v) = tanh(v) = (e^{(2v)} - 1)/(e^{(2v)} + 1)$.

Elman-RNNs [29] are the simplest of all RNNs, where recurrent nodes connections to themselves and potentailly all other hidden nodes in the same layer. Jordan-RNNs [30] have recurrent connections from output node(s) to hidden node(s). EXAMM can evolve Elman connections when the *add recurrent edge* mutation adds a recurrent edge from a simple neuron back to itself, and Jordan connections when it adds a recurrent edge from an output to a simple neuron. It generalizes these structures by allowing varying time spans (Jordan and Elmann network traditionally span only one time step) and arbitrary recurrent connections between from any pair of simple neurons or cells, i.e., cross-layer recurrent connections.

The Delta RNN (Δ -RNN) Cell: For models more complex than the simple RNN model, we looked to derive a vast set of gated neural architectures unified under a recent framework known as the Differential State Framework (DSF) [19]. A DSF neural model is essentially a composition of an inner function used to compute state proposals and an outer "mixing" function used to decide how much of a state proposal is incorporated into a slower moving state, *i.e.*, the longer term memory. These models are better equipped to handle the vanishing gradient problem induced by the difficulty of learning long-term dependencies over sequences [31]. Other models that can be derived under the DSF include the LSTM, the GRU, and the MGU [19]. One of the simplest DSF models is the Δ -RNN, which has been shown to perform competitively with more complex memory models in problems ranging from language modeling [19], [32] to image decoding [33]. With $\{\alpha, \beta_1, \beta_2, b_j, m\}$ as learnable coefficient scalars, the Δ -RNN state is defined as:

$$\begin{aligned} e_{j}^{w} &= \sum_{i \in I} w_{ij} \cdot s_{i}(t) + \sum_{r \in R,k} v_{rjk} \cdot s_{r}(t-k), \quad e_{j}^{v} = m \cdot s_{j}(t-1) \\ d_{j}^{1} &= \alpha \cdot e_{j}^{v} \cdot e_{j}^{w}, \quad d_{j}^{2} = \beta_{1} \cdot e_{j}^{v} + \beta_{2} \cdot e_{j}^{w}, \quad r_{j} = \sigma(e_{j}^{w} + b_{j}) \\ \widetilde{s}_{j}(t) &= \phi_{s}(d_{j}^{1} + d_{j}^{2}), \quad s_{j}(t) = \Phi_{s}((1-r_{j}) \cdot \widetilde{s}_{j}(t) + r_{j} \cdot s_{j}(t-1)) \end{aligned}$$

The Long-Short Term Memory (LSTM): The LSTM [15] is one of the most commonly used gated neural model when modeling sequential data. The original motivation behind the LSTM was to implement the "constant error carousal" in order to mitigate the problem of vanishing gradients. This means that long-term memory can be explicitly represented with a separate cell state c_t .

The LSTM state function (without extensions, such as "peephole" connections) is implemented as follows:

$$\begin{split} f_j &= \sigma(\sum_{i \in I} w_{ij}^f \cdot s_i(t) + \sum_{r \in R,k} v_{rjk}^f \cdot s_r(t-k)) \\ i_j &= \sigma(\sum_{i \in I} w_{ij}^i \cdot s_i(t) + \sum_{r \in R,k} v_{rjk}^i \cdot s_r(t-k)) \\ \widetilde{c}_j &= tanh(\sum_{i \in I} w_{ij}^c \cdot s_i(t) + \sum_{r \in R,k} v_{ijk}^c \cdot s_r(t-k)) \\ o_j &= \sigma(\sum_{i \in I} w_{ij}^o \cdot s_i(t) + \sum_{r \in R,k} v_{ijk}^o \cdot s_r(t-k)) \\ c_j(t) &= f_j(t) \cdot c_j(t-1) + i_j \cdot \widetilde{c}_j, \quad s_j(t) = o_j \cdot \phi_s(c_j(t)) \end{split}$$

where we depict the sharing of the transformation function's output across the forget (\mathbf{f}_t) , input (\mathbf{i}_t) , cell-state proposal $(\mathbf{\tilde{c}}_t)$, and output (\mathbf{o}_t) gates. The LSTM is by far the most parameter-hungry of the DSF models we explore.

The Gated Recurrent Unit (GRU): The Gated Recurrent Unit (GRU; [16]) can be viewed as an early attempt to simplify the LSTM. Among the changes made, the model fuses the LSTM input and forgets gates into a single gate, and merges the cell state and hidden state back together. The state function based on the GRU is calculated using the following equations:

$$z_j = \sigma(\sum_{i \in I} w_{ij}^z \cdot s_i(t) + \sum_{r \in R,k} v_{ijk}^z \cdot s_r(t-k))$$
(1)

$$r_j = \sigma(\sum_{i \in I} w_{ij}^r \cdot s_i(t) + \sum_{r \in R,k} v_{ijk}^r \cdot s_r(t-k))$$
(2)

$$\widetilde{s}_j(t) = \phi_s(\sum_{i \in I} w_{ij}^s \cdot s_i(t) + \sum_{r \in R,k} v_{ijk}^s \cdot (r_j \cdot s_r(t-k))$$
(3)

 $s_j(t) = z_j \cdot \widetilde{s}_j + (1 - z_j) \cdot s_j(t - 1)$ (4)

noting that $\phi_s(v) = tanh(v)$.

¹The bias is omitted for clarity and simplicity of presentation.

The Minimally-Gated Unit (MGU): The MGU model is very similar in structure to the GRU, reducing number of required parameters by merging its reset and update gates into a single forget gate [18]. The state computation proceeds as follows:

$$f_j = \sigma(\sum_{i \in I} w_{ij}^f \cdot s_i(t) + \sum_{r \in R,k} v_{ijk}^f \cdot s_r(t-k))$$
(5)

$$\widetilde{s}_j(t) = \phi_s(\sum_{i \in I} w_{ij}^s \cdot s_i(t) + \sum_{r \in R,k} v_{ijk}^s \cdot (f_j \cdot s_r(t-k))$$
(6)

$$s_j(t) = z_j \cdot \tilde{s}_j + (1 - z_j) \cdot s_j(t - 1).$$
 (7)

The Update-Gated RNN (UGRNN): The UGRNN [17] updates are defined in the following manner:

$$c_{j} = \phi_{s} \left(\sum_{i \in I} w_{ij}^{c} \cdot s_{i}(t) + \sum_{r \in R,k} v_{ijk}^{c} \cdot s_{r}(t-k) \right)$$
(8)

$$g_j = \sigma(\sum_{i \in I} w_{ij}^g \cdot s_i(t) + \sum_{r \in R,k} v_{ijk}^g \cdot s_r(t-k))$$
(9)

$$s_j(t) = g_j \cdot s_j(t-1) + (1-g_j) \cdot c_j.$$
(10)

The UGRNN, though more expensive than the Δ -RNN, is a simple model, essentially working like an Elman-RNN with a single update gate. This extra gate decides whether a hidden state is carried over from the previous time step or if the state should be updated.

IV. OPEN DATA AND REPRODUCIBILITY

This work utilizes two data sets to benchmark the memory cells and RNNs evolved by EXAM. The first comes from a selection of 10 flights worth of data from the National General Aviation Flight Information Database (NGAFID) [34] and the other comes from a coal-fired power plant (which has requested to remain anonymous). Both datasets are multivariate (26 and 12 parameters, respectively), non-seasonal, and the parameter recordings are not independent. Furthermore, they are very long – the aviation time series range from 1 to 3 hours worth of per-second data while the power plant data consists of 10 days worth of per-minute readings. These data sets are provided openly through the EXAMM GitHub repository², in part for reproducibility, but also to provide a valuable resource to the field. To the authors' knowledge, real world time series data sets of this size and at this scale are not freely available.

RPM (rotations per minute) and *pitch* were selected as prediction parameters from the aviation data since RPM is a product of engine activity, with other engine-related parameters being correlated, and since pitch is directly influenced by pilot controls, making it particularly challenging to predict. For the coal plant data, *main flame intensity* and *supplementary fuel flow* were selected as parameters of interest. Similar to the choices from the NGAFID data, main flame intensity is mostly a product of conditions within the (coal) burner, while supplementary fuel flow is more directly controlled by human operators.

A. Aviation Flight Recorder Data

With permission, data from 10 flights was extracted from the NGAFID. Each of the 10 flight data files last over an hour, and consist of per-second data recordings from 26 parameters:

- 1) Altitude Above Ground Level (AltAGL)
- 2) Engine 1 Cylinder Head Temperature 1 (E1 CHT1)
- 3) Engine 1 Cylinder Head Temperature 2 (E1 CHT2)
- 4) Engine 1 Cylinder Head Temperature 3 (E1 CHT3)
- 5) Engine 1 Cylinder Head Temperature 4 (E1 CHT4)
- 6) Engine 1 Exhaust Gas Temperature 1 (E1 EGT1)
- 7) Engine 1 Exhaust Gas Temperature 2 (E1 EGT2)
- 8) Engine 1 Exhaust Gas Temperature 3 (E1 EGT3)
- 9) Engine 1 Exhaust Gas Temperature 4 (E1 EGT4)
- 10) Engine 1 Oil Pressure (E1 OilP)
- 11) Engine 1 Oil Temperature (E1 OilT)
- 12) Engine 1 Rotations Per minute (E1 RPM)
- 13) Fuel Quantity Left (FQtyL)
- 14) Fuel Quantity Right (FQtyR)
- 15) GndSpd Ground Speed (GndSpd)
- 16) Indicated Air Speed (IAS)
- 17) Lateral Acceleration (LatAc)
- 18) Normal Acceleration (NormAc)
- 19) Outside Air Temperature (OAT)
- 20) Pitch
- 21) Roll
- 22) True Airspeed (TAS)
- 23) Voltage 1 (volt1)
- 24) Voltage 2 (volt2)
- 25) Vertical Speed (VSpd)
- 26) Vertical Speed Gs (VSpdG)

These files had identifying information (fleet identifier, tail number, date and time, and latitude/longitude coordinates) which was removed to protect the identify of the pilots. The data is provided unnormalized.

For this work, two parameters were selected as prediction targets: *RPM* and *Pitch*. These are interesting as the first (RPM) is a product of engine activity, with other engine related parameters being correlated; while Pitch is most directly influenced by pilot controls, making it particularly challenging to predict.

B. Coal-fired Power Plant Data

This data set consists of 10 days of per-minute data readings extracted from 12 of the plant's burners. Each of these 12 data files has 12 parameters of time series data:

- 1) Conditioner Inlet Temp
- 2) Conditioner Outlet Temp
- 3) Coal Feeder Rate
- 4) Primary Air Flow
- 5) Primary Air Split
- 6) System Secondary Air Flow Total
- 7) Secondary Air Flow
- 8) Secondary Air Split
- 9) Tertiary Air Split
- 10) Total Combined Air Flow

²https://github.com/travisdesell/exact

11) Supplementary Fuel Flow

12) Main Flame Intensity

In order to protect the confidentiality of the power plant which provided the data, along with any sensitive data elements, all identifying data has been scrubbed from the data sets (such as dates, times, locations and facility names). Further, the data has been pre-normalized between 0 and 1 as a further precaution. So while the data cannot be reverse engineered to identify the originating power plant or actual parameter values – it still is an extremely valuable test data set for times series data prediction as it consists of real world data from a highly complex system with interdependent data streams.

For this data set, two of the parameters were of key interest for time series data prediction, *Main Flame Intensity* and *Supplementary Fuel Flow*. Similar to the choices from the NGAFID data, Main Flame Intensity is mostly a product of conditions within the burner, while Supplementary Fuel Flow is more directly controlled by humans.

V. RESULTS

A. Computing Environment

Results were gathered using university research computing systems. Compute nodes utilized ranged between 10 core 2.3 GHz Intel®Xeon®CPU E5-2650 v3, 32 core 2.6 GHz AMD OpteronTMProcessor 6282 SE and 48 core 2.5 GHz AMD OpteronTMProcessor 6180 SEs, which was unavoidable due to cluster scheduling policies. All compute nodes ran RedHat Enterprise Linux 6.10. This did result in some variation in performance, however discrepancies in timing were overcome by averaging over multiple runs in aggregate.

B. Experimental Design

To better understand how the different memory cells performed in time series data prediction, multiple EXAMM runs were conducted that allowed different types of memory cells. The first set of runs (5) only any added cells of only a single, particular memory cell type, *i.e.*, either a Δ -RNN, GRU, LSTM, MGU, or UGRNN. The next set of runs (5) was nearly identical, except these allowed nodes to be simple neurons in addition to each particular memory cell type (such runs are appended with a +*simple* in the result tables). One final version was run where all cell types and simple neurons were allowed; resulting in 11 different EXAMM run types (such runs are labeled as all in the result tables). These different types of runs were done for each of the four prediction parameters (RPM, pitch, main flame intensity, and supplementary fuel flow). K-fold cross validation was done for each prediction parameter, with a fold size of 2. This resulted in 5 folds for the NGAFID data (as it had 10 flight data files), and 6 folds for the coal plant data (as it has 12 burner data files). Each fold and EXAMM setting run was repeated 10 times. In total, each of the 11 EXAMM run types was done 110 times (50 times for the NGAFID data k-fold validation and 60 times for the coal data k-fold validation), for a total of 2,420 separate runs.

All neural networks were trained with backpropagation and stochastic gradient descent (SGD) using the same hyperparameters. SGD was run with a learning rate $\eta = 0.001$, utilizing Nesterov momentum with mu = 0.9. No dropout regularization was used since it has been shown in other work to reduce performance when training RNNs for time series prediction [24]. To prevent exploding gradients, gradient clipping (as described by Pascanu et al. [35]) was used when the norm of the gradient was above a threshold of 1.0. To improve performance for vanishing gradients, gradient boosting (the opposite of clipping) was used when the norm of the gradient was below a threshold of 0.05. The forget gate bias of the LSTM cells had 1.0 added to it as this has been shown to vield significant improvements in training time by Jozefowicz et al. [36]; otherwise weights were initialized as described in Section III-B.

Each EXAMM run consisted of 10 islands, each with a population size of 5, and new RNNs were generated via intraisland crossover (at 20%), mutation (at 70%), and inter-island crossover at (10%). All mutation operations (described in Section III) except for *split edge* were utilized, as *split edge* can be recreated with the *add node* and *disable edge* operations. The 10 utilized mutation operations were performed each with a uniform 10% chance. Each EXAMM run generated 2000 RNNs, with each RNN being trained for 10 epochs. These runs were performed utilizing 20 processors in parallel, and on average required approximately 0.5 compute hours. In total, these results come from training over 4,840,000 RNNs, requiring ~24,200 CPU hours of compute time.

C. Evolved RNN Performance

Table I shows aggregated results for each of the 50 or 60 EXAMM runs done (5 or 6 folds, each with 10 repeats) that allowed only one of each memory cell type, along with the EXAMM runs that allowed for added nodes to be of all node types (*i.e.*, *all*). Table II further shows the aggregated results for runs which allowed both simple neurons and one particular cell type (*i.e.*, +simple).

These tables present the minimum, average, and maximum mean squared error (MSE) on average across the runs. Table I also shows the minimum, average, and maximum number of hidden nodes, which would be entirely of the one memory cell type, or of any memory cell type or simple neurons in the case of the *all* runs, as well as how the the number of nodes correlate to the MSE. Similar statistics are shown for the numbers of feed forward edges and the numbers of recurrent edges. Note that as a lower MSE is better, a negative correlation means that having more nodes or edges was correlated to lower MSE. Table II also divides hidden nodes into counts for simple neurons and number of memory cell nodes for a certain run type. Top 2 best models (avg) scores are shown in **bold**.

The best found MSE scores across the four prediction parameters had a wide range, which could be expected due to varying complexities. As such, to properly rank performance of different run types a metric was needed. Table IV orders

						Flame 1	Intensity								
		MSE]	Edges				. Edges			Hido	len Nodes	
Run Type	Min	Avg	Max	Min	Avg	Max	Corr.	Min	Avg	Max	Corr.	Min	Avg	Max	Corr.
all	0.000438215	0.00168176	0.00308112	11	28	64	-0.274	0	5.7	15	-0.147	11	16	22	-0.228
Δ -RNN	0.000419515	0.00164756	0.00365646	15	29	72	-0.163	1	6.3	16	-0.136	11	16	25	-0.251
GRU	0.000493313	0.00166337	0.0035205	11	28	56	-0.115	1	6.6	20	-0.101	12	16	21	0.109
LSTM	0.000460576	0.00174216	0.00386718	11	28	53	-0.187	0	7.9	26	-0.386	10	16	21	-0.241
MGU	0.000644687	0.00174361	0.00342526	8	27	49	-0.241	1	6.5	19	-0.127	13	16	21	-0.373
UGRNN	0.000531725	0.00166423	0.00399293	19	28	54	-0.215	1	6.9	25	-0.192	12	16	22	-0.209
						Fuel	Flow								
		MSE]	Edges			Rec	. Edges			Hide	len Nodes	
Run Type	Min	Avg	Max	Min	Avg	Max	Corr.	Min	Avg	Max	Corr.	Min	Avg	Max	Corr.
all	4.86288e-06	0.000134371	0.000293377	12	31	55	0.138	0	6.7	18	0.15	8	16	22	0.251
Δ -RNN	6.44047e-06	0.000140518	0.000306163	15	29	57	-0.0995	0	6.8	17	-0.264	12	16	22	-0.194
GRU	1.65072e-05	0.000131481	0.000289922	16	31	66	-0.193	0	6.6	29	-0.209	10	16	22	-0.146
LSTM	5.90673e-06	0.000121158	0.000262113	14	31	65	-0.0119	1	6.8	22	-0.028	11	16	23	0.0572
MGU	1.83369e-05	0.00013616	0.000281395	13	32	63	-0.235	0	6.7	17	-0.303	7	16	23	-0.141
UGRNN	6.08753e-06	0.000143734	0.000341757	10	29	78	0.0308	0	6.7	19	-0.0971	9	16	22	0.158
						RI	PM								
		MSE]	Edges			Rec	. Edges			Hide	len Nodes	
Run Type	Min	Avg	Max	Min	Avg	Max	Corr.	Min	Avg	Max	Corr.	Min	Avg	Max	Corr.
all	0.00410034	0.00725003	0.0142856	25	36	54	0.215	0	5.3	13	-0.133	24	29	33	0.0921
Δ -RNN	0.00247046	0.0066479	0.0132178	23	37	66	0.00237	0	4.8	16	-0.083	25	29	35	-0.00169
GRU	0.00221411	0.00684006	0.0108375	24	36	72	-0.207	0	5.9	16	-0.342	25	29	36	-0.262
LSTM	0.00323368	0.00736006	0.014325	24	35	46	0.018	0	4.7	14	-0.437	23	28	30	-0.14
MGU	0.00343788	0.00730802	0.0153965	28	38	57	0.057	0	5.3	14	-0.519	25	29	33	0.124
UGRNN	0.00307084	0.00716817	0.0116206	25	36	66	-0.277	0	5.3	15	-0.373	25	29	36	-0.315
						Pi	tch								
		MSE]	Edges			Rec	. Edges			Hido	len Nodes	
Run Type	Min	Avg	Max	Min	Avg	Max	Corr.	Min	Avg	Max	Corr.	Min	Avg	Max	Corr.
all	0.00101445	0.00347982	0.00582811	22	35	67	-0.132	0	3	10	-0.223	24	28	35	-0.109
Δ -RNN	0.00149607	0.00328248	0.00557884	24	35	49	0.0524	0	3.4	12	-0.114	23	28	32	0.104
GRU	0.00133482	0.00323731	0.00506138	22	34	54	-0.0691	0	3	11	-0.212	24	28	32	-0.113
LSTM	0.00114187	0.0033505	0.00558931	26	35	54	-0.147	0	3.5	9	-0.126	24	28	33	-0.117
MGU	0.00148457	0.00327753	0.00565053	26	35	51	0.0707	0	3.3	11	-0.388	25	28	32	0.0442
UGRNN	0.00117033	0.0033487	0.00559272	22	33	50	-0.0981	0	3	13	-0.165	23	28	32	-0.0133

TABLE I: Statistics for RNNs Evolved With Individual Memory Cells and RNNs Evolved With All Memory Types.

								Flame In	tensity										
		MSE			E	dges			Rec	: Edges			Men	nory Cells			Simp	le Neurons	
Run Type	Min	Avg	Max	Min	Avg	Max	Corr.	Min	Avg	Max	Corr.	Min	Avg	Max	Corr.	Min	Avg	Max	Corr.
Δ -RNN+simple	0.00042112	0.0015147	0.0038971	12	27	62	-0.25	1	7.7	20	-0.222	0	1.9	7	-0.137	0	1.6	6	-0.238
GRU+simple	0.00068848	0.0016726	0.0039989	18	29	55	-0.23	0	7.1	17	-0.11	0	1.8	4	-0.133	0	1.9	6	-0.384
LSTM+simple	0.00046443	0.0015194	0.0032395	19	31	65	-0.21	1	7.3	18	-0.208	0	1.6	6	-0.27	0	2.2	7	0.0671
MGU+simple	0.00053109	0.0016088	0.003241	18	29	55	-0.19	0	7.8	21	-0.238	0	1.6	6	-0.00329	0	2	7	-0.213
UGRNN+simple	0.00043835	0.0016874	0.0041592	16	29	50	-0.31	1	6.2	16	-0.274	0	1.5	5	-0.349	0	2.4	6	-0.061

								Fuel F	low										
		MSE			E	dges			Rec	: Edges			Men	nory Cells			Simpl	le Neurons	
Run Type	Min	Avg	Max	Min	Avg	Max	Corr.	Min	Avg	Max	Corr.	Min	Avg	Max	Corr.	Min	Avg	Max	Corr.
Δ -RNN+simple	8.1558e-06	0.00012294	0.00026373	9	29	55	-0.12	0	6.4	15	-0.026	0	1.7	5	-0.2	0	1.8	6	-0.00871
GRU+simple	1.3193e-05	0.00013061	0.00028389	9	31	62	-0.17	0	6	21	0.00366	0	2.1	6	-0.196	0	1.8	6	0.0181
LSTM+simple	8.6606e-06	0.00013071	0.00025847	14	31	61	-0.098	0	6.5	22	-0.187	0	2.2	7	-0.0819	0	1.9	5	0.0817
MGU+simple	7.2707e-06	0.00012367	0.00028356	11	33	61	-0.072	1	6.8	26	-0.228	0	2.2	7	-0.107	0	2.1	8	-0.0846
UGRNN+simple	5.6189e-06	0.00014337	0.00030186	12	32	62	-0.15	0	6.1	15	-0.253	0	1.8	6	-0.0961	0	2.3	8	-0.102

								RPM	AI .										
		MSE			E	dges			Rea	: Edges			Men	nory Cells			Simp	le Neurons	
Run Type	Min	Avg	Max	Min	Avg	Max	Corr.	Min	Avg	Max	Corr.	Min	Avg	Max	Corr.	Min	Avg	Max	Corr.
Δ -RNN+simple	0.0029348	0.0069672	0.011182	25	36	48	-0.17	0	4.8	14	-0.496	0	0.88	3	0.0549	0	1.2	4	-0.142
GRU+simple	0.0039148	0.0070081	0.012816	29	38	63	-0.054	1	5.1	13	-0.049	0	1.2	4	-0.0963	0	1.3	5	0.0855
LSTM+simple	0.0027877	0.0063965	0.01091	27	37	60	-0.11	0	4.6	11	-0.5	0	1	3	-0.223	0	1.2	4	0.0936
MGU+simple	0.0024288	0.0065725	0.011325	24	36	44	0.051	0	5.6	13	-0.391	0	0.98	3	-0.0944	0	1.1	3	0.178
UGRNN+simple	0.0037114	0.0073811	0.012252	22	38	48	-0.11	0	4.3	10	-0.304	0	1.1	4	-0.00488	0	1.4	4	-0.0642

								Pitc	h										
		MSE			E	dges			Rec	. Edges			Men	nory Cells			Simp	le Neurons	;
Run Type	Min	Avg	Max	Min	Avg	Max	Corr.	Min	Avg	Max	Corr.	Min	Avg	Max	Corr.	Min	Avg	Max	Corr.
Δ -RNN+simple	0.0015138	0.0032316	0.0053377	20	34	54	-0.14	0	2.6	8	-0.341	0	0.86	4	-0.00809	0	1.1	5	-0.168
GRU+simple	0.0011566	0.0032083	0.0067072	22	36	57	0.056	0	3.2	9	0.0472	0	0.98	4	0.042	0	1	3	-0.00732
LSTM+simple	0.0010335	0.0032365	0.005414	22	34	52	-0.25	0	2.9	10	-0.233	0	0.9	3	-0.176	0	0.98	4	-0.157
MGU+simple	0.0010133	0.0033326	0.0060865	25	35	53	-0.15	0	2.8	11	-0.307	0	0.88	3	-0.276	0	0.98	5	0.163
UGRNN+simple	0.0015551	0.0031352	0.0053126	22	34	52	0.1	0	3.1	8	-0.216	0	0.94	3	-0.063	0	1	4	0.181

TABLE II: Statistics for RNNs Evolved With Simple Neurons and Memory Cells.

		Si	mple			L	STM			UC	RNN			Δ	-RNN			Ν	IGU			G	RU	
Run Type	Min	Avg	Max	Corr	Min	Avg	Max	Corr	Min	Avg	Max	Corr	Min	Avg	Max	Corr	Min	Avg	Max	Corr	Min	Avg	Max	Corr
flame	0	0.6	3	0.044	0	0.5	3	-0.26	0	0.5	4	-0.11	0	0.5	2	-0.12	0	0.5	3	0.073	0	0.7	4	-0.17
fuel flow	0	0.8	3	0.0076	0	0.6	4	0.11	0	0.7	4	-0.061	0	0.6	4	-0.041	0	0.6	2	0.092	0	0.8	3	0.018
pitch	0	0.3	3	-0.017	0	0.3	3	-0.031	0	0.3	2	0.032	0	0.2	2	0.052	0	0.5	2	-0.036	0	0.3	2	-0.32
rpm	0	0.4	2	-0.14	0	0.4	2	0.24	0	0.3	2	-0.086	0	0.4	2	0.026	0	0.4	3	0.32	0	0.2	2	-0.16

TABLE III: Hidden Node Counts and Correlations to Mean Square Error for EXAMM Runs Using All Cell Types.

n		Flame In		Worst Case					
Best Ca		Avg. Ca							
Δ-RNN	-0.92312 -0.90534	△-RNN+simple	-1.7775	all	-1.5404				
Δ -RNN+simple		LSTM+simple	-1.7148	LSTM+simple	-1.1066				
all	-0.71602	MGU+simple	-0.53749	MGU+simple	-1.1026				
UGRNN+simple	-0.71451	Δ -RNN	-0.026901	MGU	-0.59787				
LSTM	-0.46836	GRU	0.18143	GRU	-0.33703				
LSTM+simple	-0.42565	UGRNN	0.19272	Δ -RNN	0.035348				
GRU	-0.10578	GRU+simple	0.30281	LSTM	0.61246				
MGU+simple	0.31264	all	0.42371	delta+simple	0.69439				
UGRNN	0.31964	UGRNN+simple	0.49785	UGRNN	0.9569				
MGU	1.5708	LSTM	1.2196	GRU+simple	0.97318				
GRU+simple	2.0557	MGU	1.2386	UGRNN+simple	1.4123				
5110 - Silipi				• • • • • • • • • • • • •					
Best Ca		Fuel fi Avg. Ca		Worst					
all	-0.92643	LSTM	-1.4415	LSTM+simple	-1.2349				
UGRNN+simple	-0.7644	Δ -RNN+simple	-1.2172	LSTM	-1.0818				
LSTM	-0.70271	MGU+simple	-1.1255	Δ -RNN+simple	-1.014				
UGRNN	-0.66396	GRU+simple	-0.25195	MGU	-0.27097				
Δ -RNN	-0.58832	LSTM+simple	-0.23921	MGU+simple	-0.1799				
MGU+simple	-0.41037	GRU	-0.14222	GRU+simple	-0.16598				
Δ -RNN+simple	-0.22068	all	0.22163	GRU	0.087564				
LSTM+simple	-0.1125	MGU	0.44679	all 0.23284					
GRU+simple	0.85882	Δ -RNN	0.99531	UGRNN+simple 0.58938					
GRU	1.5692	UGRNN+simple	1.3537	Δ -RNN	0.77052				
MGU	1.9613	UGRNN	1.4002	UGRNN	2.2672				
		1	11						
Best Ca	150	RPN Avg. Ca		Worst	- ace				
GRU	-1.444	LSTM+simple	-1.7472	GRU	-1.0958				
MGU+simple	-1.1012	MGU+simple	-1.2299	LSTM+simple	-1.0499				
Δ -RNN	-1.0347	Δ -RNN	-1.0081	Δ -RNN+simple	-0.87687				
LSTM+simple	-0.52825	GRU	-0.4433	MGU+simple	-0.78566				
Δ -RNN+simple	-0.29348	Δ -RNN+simple	-0.069508	UGRNN	-0.59783				
UGRNN	-0.076276	GRU+simple	0.050686	UGRNN+simple	-0.19645				
LSTM	0.18368	UGRNN	0.52115	GRU+simple	0.16258				
MGU	0.50967	all	0.76179	Δ-RNN	0.41787				
UGRNN+simple	0.9463	MGU	0.93224	all	1.0968				
GRU+simple	1.271	LSTM	1.0852	LSTM	1.1219				
all	1.5672	UGRNN+simple	1.147	MGU	1.8033				
an	1.5072	Contractingie	1.147		1.0055				
		Pite			2				
Best Ca		Avg. Ca		Worst					
MGU+simple	-1.1631	UGRNN+simple	-1.6163	GRU	-1.3295				
all	-1.1577	GRU+simple	-0.82052	UGRNN+simple	-0.76284				
	-1.0698	Δ -RNN+simple	-0.56665	Δ -RNN+simple	-0.70622				
LSTM+simple									
LSTM+simple LSTM	-0.5688	LSTM+simple	-0.51389	LSTM+simple	-0.53415				
		LSTM+simple GRU	-0.51389 -0.5047	LSTM+simple ∆-RNN	-0.53415 -0.16235				
LSTM	-0.5688								
LSTM GRU+simple	-0.5688 -0.50079	GRU	-0.5047	Δ-RNN	-0.16235				
LSTM GRU+simple UGRNN GRU	-0.5688 -0.50079 -0.43726 0.32298	GRU MGU delta	-0.5047 -0.066984 -0.013118	Δ-RNN LSTM UGRNN	-0.16235 -0.13873 -0.13104				
LSTM GRU+simple UGRNN GRU MGU	-0.5688 -0.50079 -0.43726 0.32298 1.0151	GRU MGU delta MGU+simple	-0.5047 -0.066984 -0.013118 0.53287	Δ-RNN LSTM UGRNN MGU	-0.16235 -0.13873 -0.13104 -0.00065639				
LSTM GRU+simple UGRNN GRU MGU △-RNN	-0.5688 -0.50079 -0.43726 0.32298 1.0151 1.0682	GRU MGU delta MGU+simple UGRNN	-0.5047 -0.066984 -0.013118 0.53287 0.70761	Δ-RNN LSTM UGRNN MGU all	-0.16235 -0.13873 -0.13104 -0.00065639 0.39991				
LSTM GRU+simple UGRNN GRU MGU Δ-RNN Δ-RNN+simple	-0.5688 -0.50079 -0.43726 0.32298 1.0151 1.0682 1.1501	GRU MGU delta MGU+simple UGRNN LSTM	-0.5047 -0.066984 -0.013118 0.53287 0.70761 0.72719	∆-RNN LSTM UGRNN MGU all MGU+simple	-0.16235 -0.13873 -0.13104 -0.00065639 0.39991 0.98284				
LSTM GRU+simple UGRNN GRU MGU Δ-RNN	-0.5688 -0.50079 -0.43726 0.32298 1.0151 1.0682	GRU MGU delta MGU+simple UGRNN	-0.5047 -0.066984 -0.013118 0.53287 0.70761	Δ-RNN LSTM UGRNN MGU all	-0.16235 -0.13873 -0.13104 -0.00065639 0.39991				
LSTM GRU+simple UGRNN GRU MGU Δ-RNN Δ-RNN+simple UGRNN+simple	-0.5688 -0.50079 -0.43726 0.32298 1.0151 1.0682 1.1501 1.3411	GRU MGU delta MGU+simple UGRNN LSTM all Overall Co	-0.5047 -0.066984 -0.013118 0.53287 0.70761 0.72719 2.1345 mbined	Δ-RNN LSTM UGRNN MGU all MGU+simple GRU+simple	-0.16235 -0.13873 -0.13104 -0.00065639 0.39991 0.98284 2.3828				
LSTM GRU+simple UGRNN GRU MGU Δ-RNN Δ-RNN+simple UGRNN+simple	-0.5688 -0.50079 -0.43726 0.32298 1.0151 1.0682 1.1501 1.3411	GRU MGU delta MGU+simple UGRNN LSTM all Overall Co Avg. Ca	-0.5047 -0.066984 -0.013118 0.53287 0.70761 0.72719 2.1345 mbined se	△-RNN LSTM UGRNN MGU all MGU+simple GRU+simple	-0.16235 -0.13873 -0.13104 -0.00065639 0.39991 0.98284 2.3828 Case				
LSTM GRU+simple UGRNN GRU MGU Δ-RNN Δ-RNN+simple UGRNN+simple Best Ca MGU+simple	-0.5688 -0.50079 -0.43726 0.32298 1.0151 1.0682 1.1501 1.3411	GRU MGU delta MGU+simple UGRNN LSTM all Overall Co Avg. Ca LSTM+simple	-0.5047 -0.066984 -0.013118 0.53287 0.70761 0.72719 2.1345 mbined se -1.0538	△-RNN LSTM UGRNN MGU all MGU+simple GRU+simple Vorst (LSTM+simple	-0.16235 -0.13873 -0.13104 -0.00065639 0.39991 0.98284 2.3828 Case -0.98141				
LSTM GRU+simple UGRNN GRU MGU Δ-RNN Δ-RNN+simple UGRNN+simple Best Cr MGU+simple LSTM+simple	-0.5688 -0.50079 -0.43726 0.32298 1.0151 1.0682 1.1501 1.3411 ise -0.59051 -0.53405	GRU MGU delta MGU+simple UGRNN LSTM all Overall Co Avg. Ca LSTM+simple	-0.5047 -0.066984 -0.013118 0.53287 0.70761 0.72719 2.1345 mbined se -1.0538 -0.90771	△-RŇN LSTM UGRNN MGU all MGU+simple GRU+simple LSTM+simple GRU	-0.16235 -0.13873 -0.13104 -0.00065639 0.39991 0.98284 2.3828 Case				
LSTM GRU+simple UGRNN GRU MGU Δ-RNN Δ-RNN+simple UGRNN+simple Best Ca MGU+simple	-0.5688 -0.50079 -0.43726 0.32298 1.0151 1.0682 1.1501 1.3411	GRU MGU delta MGU+simple UGRNN LSTM all Overall Co Avg. Ca LSTM+simple	-0.5047 -0.066984 -0.013118 0.53287 0.70761 0.72719 2.1345 mbined se -1.0538 -0.90771	△-RNN LSTM UGRNN MGU all MGU+simple GRU+simple Vorst (LSTM+simple	-0.16235 -0.13873 -0.13104 -0.00065639 0.39991 0.98284 2.3828 Case -0.98141				
LSTM GRU+simple UGRNN GRU MGU Δ-RNN Δ-RNN+simple UGRNN+simple Best Cr MGU+simple LSTM+simple LSTM	-0.5688 -0.50079 -0.43726 0.32298 1.0151 1.0682 1.1501 1.3411 use -0.59051 -0.59051 -0.53405 -0.38905	GRU MGU delta MGU+simple UGRNN LSTM all Overall Co Avg. Ca LSTM+simple	-0.5047 -0.066984 -0.013118 0.53287 0.70761 0.72719 2.1345 mbined se -1.0538 -0.90771 -0.59001	△-RŇN LSTM UGRNN MGU all MGU+simple GRU+simple LSTM+simple GRU GRU LSTM+simple	-0.16235 -0.13873 -0.13104 -0.00065639 0.39991 0.98284 2.3828 -0.98141 -0.6687 -0.98141				
LSTM GRU+simple UGRNN GRU MGU Δ-RNN Δ-RNN+simple UGRNN+simple LSTM+simple LSTM-simple LSTM	-0.5688 -0.50079 -0.43726 0.32298 1.0151 1.0682 1.1501 1.3411 ise -0.59051 -0.3405 -0.38905 -0.36948	GRU MGU delta MGU+simple UGRNN LSTM all Overall Cα Avg. Ca LSTM+simple Δ-RNN+simple MGU+simple GRU	-0.5047 -0.066984 -0.013118 0.53287 0.70761 0.72719 2.1345 mbined se -1.0538 -0.90771 -0.59001 -0.2272	∆-RNN LSTM LSTM UGRNN MGU all MGU+simple GRU+simple LSTM+simple GRU △-RNN+simple MGU+simple	-0.16235 -0.13873 -0.13104 -0.00065639 0.39991 0.98284 2.3828 Case -0.98141 -0.6687 -0.47566 -0.27133				
LSTM GRU+simple UGRNN GRU MGU Δ-RNN MOU-Simple UGRNN-simple LSTM-simple LSTM-simple LSTM-all all	-0.5688 -0.50079 -0.43726 0.32298 1.0151 1.0682 1.1501 1.3411 see -0.59051 -0.38905 -0.36948 -0.306948 -0.30824	GrkU MGU delta MGU+simple UGRNN LSTM all Overall Co ∆-RNN+simple MGU+simple GRU	-0.5047 -0.066984 -0.013118 0.53287 0.70761 0.72719 2.1345 mbined se -1.0538 -0.90771 -0.59001 -0.2272 -0.17974	Δ-RŇN LSTM UGRNN MGU all MGU+simple GRU+simple LSTM+simple GRU Δ-RNN+simple MGU+simple	-0.16235 -0.13873 -0.13104 -0.00065639 0.39991 0.98284 2.3828 -0.98141 -0.6687 -0.47566 -0.27133 0.047292				
LSTM GRU+simple UGRNN GRU MGU A-RNN-simple UGRNN+simple LSTM+simple LSTM A-RNN all UGRNN	-0.5688 -0.50079 -0.43726 0.32298 1.0151 1.0682 1.1501 1.3411 se -0.59051 -0.33405 -0.38905 -0.36948 -0.30824 -0.21446	GRU MGU delta MGU+simple UGRNN LSTM all Overall Cc Avg. Ca LSTM+simple △-RNN+simple GRU GRU+simple A-RNN	-0.5047 -0.066984 -0.013118 0.53287 0.70761 0.72719 2.1345 mbined se -1.0538 -0.90771 -0.59001 -0.2272 -0.17974 -0.013211	△-RŇN LSTM UGRNN MGU all MGU+simple GRU+simple GRU LSTM+simple GRU △-RNN+simple MGU+simple all LSTM	-0.16235 -0.13873 -0.13104 -0.00065639 0.39991 0.98284 2.3828 -0.98141 -0.6687 -0.47566 -0.47566 -0.47566 -0.47756				
LSTM GRU+simple UGRNN GRU MGU ∆-RNN korkn+simple UGRNN+simple LSTM A-RNN all UGRNN ∆-RNN A-RNN A	-0.5688 -0.50079 -0.43726 0.32298 1.0151 1.0682 1.1501 1.3411 ise -0.59051 -0.53405 -0.38905 -0.38905 -0.38948 -0.0824 -0.21446 -0.067358	GrkU MGU delta MGU+simple UGRNN LSTM all Overall Co Avg. Ca LSTM+simple ∆-RNN+simple GRU GRU+simple △-RNN UGRNN+simple	-0.5047 -0.066984 -0.013118 0.53287 0.70761 0.72719 2.1345 mbined se -1.0538 -0.90771 -0.59001 -0.2272 -0.17974 -0.013211 0.34556	Δ-RŃN LSTM UGRNN MGU all MGU+simple GRU+simple CRU Δ-RNN+simple MGU+simple MGU+simple MGU+simple MGU-MGU	-0.16235 -0.13873 -0.13104 -0.00065639 0.39991 0.98284 2.3828 -0.98141 -0.6687 -0.47566 -0.27133 0.047292 0.12847 0.23345				
LSTM GRU+simple UGRNN GRU MGU Δ-RNN Δ-RNN-simple UGRNN-simple LSTM Δ-RNN all UGRNN all UGRNN all GRU	-0.5688 -0.50079 -0.43726 0.32298 1.0151 1.0682 1.1501 1.3411 see -0.59051 -0.53405 -0.38905 -0.36948 -0.30824 -0.21446 -0.067358 0.0885614	GrkU MGU delta MGU+simple UGRNN LSTM all Overall Co Avg. Ca LSTM+simple △-RNN+simple GRU GRU-simple GRU GRU-simple Δ-RNN UGRNN+simple	-0.5047 -0.066984 -0.013118 0.53287 0.70761 0.72719 2.1345 mbined se -1.0538 -0.90771 -0.59001 -0.2272 -0.17974 -0.013211 0.34556 0.39761	△-RNN LSTM UGRNN MGU all MGU+simple GRU+simple GRU LSTM+simple GRU all LSTM MGU+simple all LSTM MGU Sagarta	-0.16235 -0.13873 -0.13104 -0.00065639 0.39991 0.98284 2.3828 -0.98141 -0.6687 -0.47566 -0.27133 0.047292 0.12847 0.23345 0.26059				
LSTM GRU+simple UGRNN GRU MGU- MGU-simple UGRNN+simple LSTM-simple LSTM-simple LSTM-simple GRU GRNN-simple GRU UGRNN+simple	-0.5688 -0.50079 -0.43726 0.32298 1.0151 1.0682 1.1501 1.3411 -0.59051 -0.38905 -0.38905 -0.38948 -0.30824 -0.21446 -0.67358 0.0885614 0.20212	GrU MGU delta MGU+simple UGRNN LSTM all Overall Co Arg. Ca LSTM+simple ∆-RNN+simple GRU+simple ∆-RNN UGRNN+simple LSTM MGU	-0.5047 -0.066984 -0.013118 0.53287 0.70761 0.72719 2.1345 mbined se -1.0538 -0.90771 -0.59001 -0.2272 -0.17974 -0.013211 0.34556 0.39761 0.63765	Δ-RNN LSTM UGRNN all MGU+simple GRU+simple GRU LSTM+simple GRU Δ-RNN+simple MGU+simple all LSTM MGU UGRNN+simple Δ-RNN	-0.16235 -0.13873 -0.13104 -0.00065639 0.39991 0.98284 2.3828 -0.98141 -0.6687 -0.47566 -0.27133 0.047292 0.12847 0.23345 0.23345 0.26059 0.2655				
LSTM GRU+simple UGRNN GRU MGU Δ-RNN Δ-RNN-simple UGRNN-simple LSTM Δ-RNN all UGRNN all UGRNN all GRU	-0.5688 -0.50079 -0.43726 0.32298 1.0151 1.0682 1.1501 1.3411 see -0.59051 -0.53405 -0.38905 -0.36948 -0.36948 -0.21446 -0.067358 0.085614	GrkU MGU delta MGU+simple UGRNN LSTM all Overall Co Avg. Ca LSTM+simple △-RNN+simple GRU GRU-simple GRU GRU-simple Δ-RNN UGRNN+simple	-0.5047 -0.066984 -0.013118 0.53287 0.70761 0.72719 2.1345 mbined se -1.0538 -0.90771 -0.59001 -0.2272 -0.17974 -0.013211 0.34556 0.39761	△-RNN LSTM UGRNN MGU all MGU+simple GRU+simple GRU LSTM+simple GRU all LSTM MGU+simple all LSTM MGU Sagarta	-0.16235 -0.13873 -0.13104 -0.00005639 0.39991 0.98284 2.3828 -0.98284 2.3828 -0.6687 -0.27133 0.047292 0.12847 0.23345 0.26059				

TABLE IV: EXAMM Run Types Prediction Error Ranked By Standard Deviation From Mean. the 11 different run types by how many standard deviations

the 11 different run types by how many standard deviations the results were from the mean for each prediction parameter. It also provides combined rankings, averaging the deviation from the mean across the four prediction parameters. Each of these tables are ordered from best to worst – a negative deviation from the mean is that many standard deviations less than the average MSE, and lower MSE is better.

The results of these experiments led to some interesting findings which the authors feel can help inform further development of neuro-evolution algorithms as well as RNN memory cells. Many of these findings can also serve as warnings to those looking to train well performing RNNs for time series prediction. We summarize the main takeaways from these results as follows:

a) No memory structure was truly the best:: In the overall rankings (Table IV), the Δ -RNN, LSTM, and MGU cells seemed to have the highest rankings for best overall performance as well as average and worst case performance, with GRU cells being slightly behind. While UGRNN nodes

did not do so well in the total ranking, it should be noted that when coupled with simple neurons, they did perform 2nd best for the best case for fuel flow, and were the best for the average case in predicting pitch. This highlights the importance of testing a wide selection of memory cell types when developing RNNs, as there is *no free lunch* in machine learning – each memory cell type had its own strengths and weaknesses. It is valuable to note that one of the simplest memory cell types, *i.e.*, the Δ -RNN, performs consistently and competitively with the more complicated, multi-gate LSTM (at the top of the rankings), which is consistent with a growing body of results [19], [32], [33].

b) Adding simple neurons generally helped - with some notable exceptions:: When looking at the overall rankings, when simple neurons were added as an option to the neuroevolution process, the networks performed better. The only exception to this was GRU cells, which tended to perform worse when simple neurons were allowed. These results may indicate that these memory cells lack the capability to track some kinds of dependencies which the additional simple neurons make up for; this means that there is potentially room to improve these cell structures to capture whatever the simple neurons were providing. Further examination of why the GRU cells performed worse with simple neurons compared to the other memory cells may help determine the cause of this and makes for an interesting direction for future RNN work.

Another very interesting finding was that utilizing simple neurons with MGU cells resulted in a dramatic improvement, bringing them from some of the worst rankings to some of the best rankings (*e.g.*, in the overall rankings for best found networks, MGU cells alone performed the worst while MGU and simple neurons performed the best). Other cell types (LSTM and Δ -RNN) showed less of an improvement. This finding may highlight that the MGU cells could stand to benefit from further development. This should serve as a warning to others developing neuro-evolution algorithms, in that even the rather simple change of allowing simple neurons can result in significant changes in RNN predictive ability. Selection of node and cell types for neuro-evolution should be done carefully.

c) Allowing all memory cells has risks and benefits:: The authors had hoped that allowing the neuro-evolution process to simply select from all of the memory cell types would allow it to find and utilize whichever cells were most suited to the prediction problem. Unfortunately, this was not always the case. While using all memory cell types generally performed better than the mean on the best case, in the average and worst cases, it performed worse than the mean. This was most likely due to the fact that whenever a node was added to the network it could have been from any of the 6 types, so choosing from them uniformly at random ended up sometimes selecting the nodes not best to the task (as they improved the population, but not as well as another memory cell choice could have done). These results are backed up by Table III, which shows the correlations between node types and MSE (again, a negative correlation means more of that cell type resulted in a lower/better MSE), as well as the min, average and max memory cell counts among the networks found by EXAMM.

That being said, allowing all cell types did find the best networks in the case of fuel flow, 2nd best in the case of pitch, and 3rd best in the case of flame intensity; which is impressive given the larger search space for the neuroevolution process. We expect to further improve this result by dynamically adapting the rates at which memory cells are generated in part based on how well they have improved the population in the past (with caveats described in the next point) – this stands as future work which can make EXAMM an even stronger option for generating RNNs, especially in the average and worse cases where they did not fare as well.

d) Larger networks tended to perform better, yet memory cell count correlation to MSE was not a great indicator of which cells performed the best:: This last point raises some significant challenges for developing neuro-evolution algorithms. When looking at Table III and examining the memory cells types most correlated to improved performance against the memory cell types most frequently selected by EXAMM, meant that EXAMM was not selecting cell types that would produce the best performing RNNs. This may due to the fact that, in some cases, an RNN with a small number of well trained memory cells was sufficient to yield good predictions, and adding more cells to the network only served to confuse the predictions.

The implications of this are two fold: 1), running a neuroevolution strategy allowing all memory cell types and then utilizing counts or correlations to select a single memory cell type for future runs may not produce the best results, and 2), dynamically tuning which memory cells are selected by a neuro-evolution strategy is more challenging since the process may not select the best cell types (*e.g.*, when the network already has enough memory cells) – so this would at least need to be coupled with another strategy to determine when the network is "big enough".

VI. CONCLUSIONS AND FUTURE WORK

This work introduced a new neuro-evolution algorithm, Evolutionary eXploration of Augmenting Memory Models (EXAMM), for evolving recurrent neural architectures by directly incorporating powerful memory cells such as the Δ -RNN, MGU, GRU, LSTM and UGRNN units into the evolutionary process. EXAMM was evaluated on the task of predicting 4 different parameters from two large, real world time-series datasets. By using repeated k-fold cross validation and high performance computing, enough RNNs were evolved to be rigorously analyzed – a methodology the authors think should be highlighted as novel. Instead of utilizing them to outperform other algorithms on benchmarks, neuro-evolutionary processes can be used as selection methodologies, providing deeper insights into what neural structures perform best on certain tasks.

Key findings from this work show that a neuro-evolution strategy that selects from a wide number of memory cell

structures can yield performant architectures. However, it does so at the expense of reliability in the average and worst cases. Furthermore, a simple modification to the evolutionary process, *i.e.*, allowing simple neurons, can have dramatic effects on network performance. In general, while this largely benefits most memory cells, outlier cases showed wide swings from worst to best and best to worst performance. The authors hope that these results will guide future memory cell development, as the addition of simple neurons dramatically improved MGU performance, but also decreased GRU performance. Understanding cases like that of the GRU could yield improvements in cell design. Results showed that cell selection does not necessarily correlate well to the best cell types for a particular problem, partly due to the fact that good cells may not necessarily require a large network. These results should serve as cautionary information for future development of neuroevolution algorithms.

This paper opens up many avenues of future work. This includes extending EXAMM's search process to allow cellular elements of the underlying neural model to be evolved (as in Rawal and Miikulainen [22]); evolving over a large set of post-activation functions; allowing for stochastic operations, e.g., Bernoulli sampling; and incorporating operators such as convolution (to handle video sequences/time series) or its simple approximation, the perturbative operator [37]. Additional future work will involve various hyperparameter optimization strategies to dynamically determine RNN training metaparameters as well as what probabilities EXAMM uses to choose memory cell structures and what probabilities it uses for the mutation and recombination operators. Lastly, implementing even larger scale mutation operations, such as multinode/layer mutations could potentially speed up EXAMM's neuro-evolution process even further.

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