Full-Diversity Precoding Design of Bit-Interleaved Coded Multiple Beamforming with Orthogonal Frequency Division Multiplexing

Boyu Li and Ender Ayanoglu

Center for Pervasive Communications and Computing
Department of Electrical Engineering and Computer Science
The Henry Samueli School of Engineering
University of California - Irvine
Irvine, California 92697-2625
Email: boyul@uci.edu, ayanoglu@uci.edu

***Abstract*—For Multiple-Input Multiple-Output (MIMO) systems with frequency selective fading channels, Bit-Interleaved Coded Multiple Beamforming (BICMB) with Orthogonal Frequency Division Multiplexing (OFDM) can offer both spatial and multipath diversity, making it an important technique. It was proved that full diversity of BICMB-OFDM can be achieved with a condition of $R_cSL \leq 1$, where R_c, S, and L are the code rate, the number of parallel streams at each subcarrier, and the number of channel taps, respectively. In this paper, the precoding technique is applied to overcome the full diversity restriction with the minimum achievable decoding complexity.**

I. Introduction

In a MIMO system, beamforming exploiting Singular Value Decomposition (SVD) can be applied to improve the data rate or the performance, when the channel state information is known by both the transmitter and receiver [1].

For flat fading MIMO channels, beamforming with only one spatial channel achieves full diversity [2]. In addition, beamforming with more than one spatial channel without channel coding results in full diversity loss. To combat the degradation, interleaving the coded bit codeword through multiple subchannels was proposed and was called BICMB [3], [4]. BICMB can achieve full diversity as long as the code rate R_c and the number of employed subchannels S satisfy the condition $R_cS \leq 1$ [5], [6]. Other than channel coding, when a constellation precoding technique is used in uncoded and coded beamforming, this full diversity condition can be overridden [7], [8]. Specifically, without channel coding, full diversity requires that all spatial channels are precoded. In the case of precoding with BICMB, partial precoding can achieve full diversity. Partial precoding is desirable because it reduces the high decoding complexity of a fully precoded BICMB system. These precoders result in a system that achieves full diversity even without satisfying the condition $R_cS \leq 1$.

If the MIMO channel is in frequency selective fading, BICMB-OFDM can be applied to combat the inter-symbol interference caused by multipath propagation and achieve both spatial and multipath diversity [3]. OFDM is well-suited for broadband data transmission and has been selected as the air interface for IEEE 802.11 WiFi, IEEE 802.16 WiMAX, and 3GPP LTE [9]. Although more sophisticated codes, such as turbo codes and LDPC codes [10], are employed by some of the more modern standards [11], than convolutional codes employed by BICMB, convolutional codes are still very important because they can be analyzed and there are many legacy products using them. Therefore, BICMB-OFDM is an important technique for broadband wireless communication. The diversity analysis of BICMB-OFDM was carried out in [12], [13]. It was proved that full diversity of BICMB-OFDM can be achieved as long as the condition $R_cSL \leq 1$ is satisfied, where S is the number of streams transmitted at each subcarrier and L is the number of channel taps. The full diversity condition implies that keeping full diversity, increasing S may not improve the total transmission rate as R_c might need to decrease, which is a similar issue to the full diversity restriction $R_cS \leq 1$ of BICMB for flat fading MIMO channels [5], [6]. Since precoding techniques have been successfully used to solve the full diversity restriction $R_cS \leq 1$ for BICMB of flat fading MIMO channels [7], [8], they may also be applied for BICMB-OFDM to solve the full diversity restriction $R_cSL \leq 1$ for frequency selective fading MIMO channels. However, it cannot be generalized straightforwardly because of the increased system complexity.

In this paper, the precoding technique is employed to solve the full diversity restriction $R_cSL \leq 1$ for BICMB-OFDM with the minimum achievable decoding complexity. The remainder of this paper is organized as follows: Section II briefly describes BICMB-OFDM. In Section III, the system model is outlined. In Section IV, the diversity analysis is carried out, and a full diversity condition related to the combination of the precoding matrix, the convolutional code, and the bit interleaver is provided. Then, Section V develops the full-diversity precoding design with the minimum achievable decoding diversity. In Section VI, simulation results are provided. Finally, a conclusion is drawn in Section VII.

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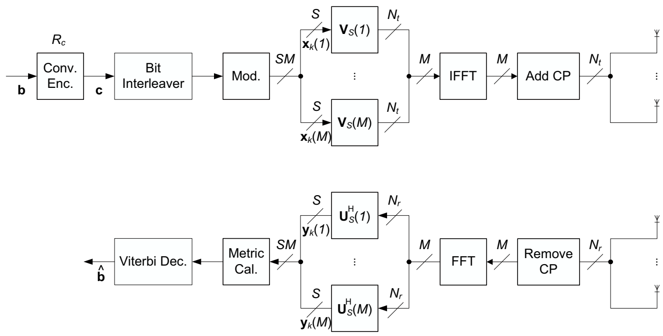

Fig. 1. Structure of BICMB-OFDM.

II. Background Knowledge of BICMB-OFDM

In this section, a brief description of BICMB-OFDM is offered as background knowledge for the following sections. More details of BICMB-OFDM can be found in [12], [13].

For frequency selective MIMO channels, BICMB-OFDM was proposed to provide both spatial diversity and multipath diversity [3]. Fig. 1 presents the structure of BICMB-OFDM. First, the bit codeword $\mathbf{c}$ is generated from the information bits by the convolutional encoder of code rate R_c, which is possibly combined with a perforation matrix for a high rate punctured code. After that, a random bit interleaver is applied to generate an interleaved bit sequence, which is then modulated, e.g., QAM, to a symbol sequence. The number of transmit and receive antennas are denoted by N_t and N_r respectively. Assume that M subcarriers are employed, and $S \leq \min\{N_t, N_r\}$ parallel streams are realized by SVD in the frequency domain for each subcarrier. Hence, an $S \times 1$ symbol vector $\mathbf{x}_k(m)$ is carried on the mth subcarrier at the kth time instant with $m = 1, \ldots, M$. The length of Cyclic Prefix (CP), which is employed for OFDM to combat ISI caused by multipath propagation, is assumed to be L_{cp} where $L_{cp} \geq L$ with L denoting the number of channel taps.

The frequency selective fading MIMO channel is assumed to be Rayleigh quasi-static and known by both the transmitter and receiver, denoted by $\breve{\mathbf{H}}(l)$ with $l = 1, \ldots, L$. Let

$$\mathbf{H}(m) = \sum_{l=1}^{L} \breve{\mathbf{H}}(l) \exp\left(-i \frac{2\pi(m-1)\tau_l}{MT}\right) \quad (1)$$

denote the quasi-static flat fading MIMO channel observed at the mth subcarrier, where T denotes the sampling period, τ_l indicates the lth tap delay, and $i = \sqrt{-1}$ [14]. The beamforming matrices at the mth subcarrier are derived by SVD of $\mathbf{H}(m)$, i.e., $\mathbf{H}(m) = \mathbf{U}(m)\mathbf{\Lambda}(m)\mathbf{V}^H(m)$, where $\mathbf{U}(m)$ and $\mathbf{V}(m)$ are unitary, and $\mathbf{\Lambda}(m)$ is a rectangular diagonal matrix whose sth diagonal element, $\lambda_s(m)$, is a singular value of $\mathbf{H}(m)$, which is positive and real, in decreasing order with $s = 1, \ldots, S$. The first S columns of $\mathbf{U}(m)$ and $\mathbf{V}(m)$, i.e., $\mathbf{U}_S(m)$ and $\mathbf{V}_S(m)$, are chosen as beamforming matrices at the receiver and transmitter for the mth subcarrier respectively.

For each subcarrier, the multiplications with beamforming matrices $\mathbf{V}_S(m)$ and $\mathbf{U}_S^H(m)$ are carried out at each subcarrier before executing IFFT and adding CP at the transmitter, and after executing FFT and removing CP at the receiver, respectively. Therefore, the input-output relation of BICMB-OFDM for the mth subcarrier at the kth time instant is

$$y_{s,k}(m) = \lambda_s(m) x_{s,k}(m) + n_{s,k}(m), \quad (2)$$

for $s = 1, \ldots, S$, where $y_{s,k}(m)$ and $x_{s,k}(m)$ are the sth element of the $S \times 1$ received symbol vector $\mathbf{y}_k(m)$ and the transmitted symbol vector $\mathbf{x}_k(m)$ respectively, and $n_{s,k}(m)$ is the additive white Gaussian noise with zero mean and variance $N_0 = N_t/\gamma$ with γ denoting the received SNR over all the receive antennas. Note that the total transmitted power is scaled by N_t in order to make the received SNR γ.

The location of the coded bit $c_{k'}$ within the transmitted symbol is denoted as $k' \rightarrow (k, m, s, j)$, meaning that $c_{k'}$ is mapped onto the jth bit position on the label of $x_{s,k}(m)$. Let χ denote the signal set of the modulation scheme, and let χ_b^j denote a subset of χ whose labels have $b \in \{0, 1\}$ at the jth bit position. By using the location information and the input-output relation in (2), the receiver calculates the Maximum Likelihood (ML) bit metrics for $c_{k'} = b$ as

$$\Delta[y_{s,k}(m), c_{k'}] = \min_{x \in \chi^j_{c_{k'}}} |y_{s,k}(m) - \lambda_s(m) x|^2 . \quad (3)$$

Finally, the ML decoder applies the soft-input Viterbi decoding to find a codeword with the minimum sum weight and its corresponding information bit sequence $\hat{\mathbf{b}}$ as

$$\hat{\mathbf{c}} = \arg\min_{\mathbf{c}} \sum_{k'} \Delta[y_{s,k}(m), c_{k'}]. \quad (4)$$

In [12], [13], the maximum achievable diversity of BICMB-OFDM was derived and the full diversity restriction of $R_c S L \leq 1$ was proved. In addition, the performance degradation due to subcarrier correlation was investigated, which showed that although the diversity is the same when SNR is relatively high, strong subcarrier correlation can result in significant performance loss for SNRs in the practical range.

In order to combat the performance loss of BICMB-OFDM due to subcarrier correlation, the subcarrier grouping technique was employed in [12], [13]. Instead of transmitting one stream of information through all subcarriers of OFDM, subcarrier grouping technique transmits multiple streams of information through multiple group of subcarriers. For BICMB-OFDM with Subcarrier Grouping (BICMB-OFDM-SG), assuming that $G = M/L$ is an integer, then G streams of bit codewords are carried on G different groups of L uncorrelated or weakly correlated subcarriers separately. Compared to no grouping, BICMB-OFDM-SG achieves better performance with the same data rate and decoding complexity while offering multi-user compatibility. More details can be found in [12], [13].

III. System Model of Precoded BICMB-OFDM-SG

As discussed in [12], [13] and Section II, BICMB-OFDM-SG is a better choice than BICMB-OFDM without subcarrier grouping. Hence, the precoding technique discussed in this paper is employed on top of BICMB-OFDM-SG. Since the G

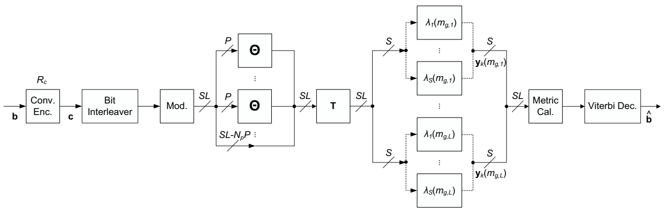

Fig. 2. Structure of BICMB-OFDM-SG with precoding in the frequency domain for one bit stream transmission of the gth subcarrier group.

groups of bit streams are carried separately in the frequency domain for BICMB-OFDM-SG and the only difference is the associated singular values of subchannels as shown in [12], [13], the precoding technique can be applied to each subcarrier group independently. Therefore, it is sufficient to consider one subcarrier group to illustrate the system model.

Fig. 2 presents the structure of precoded BICMB-OFDM-SG in the frequency domain for one bit stream transmission of the gth subcarrier group with $g = \{1, \ldots, G\}$. Compared to no precoding in [12], [13], the channel coding, bit interleaver, and modulation remain the same, while two more blocks are added at the transmitter. Specifically, $\mathbf{\Theta}$ is a $P \times P$ precoding matrix where $P \le SL$ denotes the dimension of $\mathbf{\Theta}$. The P precoded subchannels are defined as one precoded subchannel set. Let N_p denote the number of precoded subchannel sets applied for the gth subcarrier group, where $N_pP \le SL$. Hence, N_pP subchannels are precoded while the remaining $N_n = SL - N_pP$ subchannels are non-precoded. The selection of precoded subchannels is predefined by a permutation matrix $\mathbf{T}$.

Note that there are L subcarriers of each group and each subcarrier has S subchannels realized by SVD. For the sth subchannel at the lth subcarrier of the gth group, its singular value is $\lambda_s(m_{g,l})$ where $m_{g,l} = (l-1)G + g$ is the subcarrier index. Since the G subcarrier groups are independent, the group index is omitted for brevity, and $\lambda_s(m_{g,l})$ is rewritten as $\lambda_{l,s}$ where $\{l, s\}$ denotes the sth subchannel at the lth subcarrier. For the sake of convenience, the two-dimensional index is converted to a single dimensional one based on the rule $\{l, s\} \to q = (l-1)S + s$ with $q \in \{1, \ldots, SL\}$, and the inverse conversion is $q \to \{l, s\} = \{[\lfloor (q-1)/S \rfloor + 1], [(q-1) \bmod S + 1]\}$. Define $\boldsymbol{\eta}^z = [\eta_1^z \ \ldots \ \eta_P^z]$ as a vector whose elements η_p^z denote the subchannel indices of the zth precoded subchannel set with $z \in \{1, \ldots, N_p\}$, and are ordered increasingly such that $\eta_u^z < \eta_v^z$ for $u < v$. In the same way, $\boldsymbol{\omega} = [\omega_1 \ \ldots \ \omega_{N_n}]$ is defined as an increasingly ordered vector whose elements are the indices of the non-precoded subchannels.

At the kth time instant, the serial-to-parallel converter of the transmitter organizes the $SL \times 1$ transmitted symbol vector as $\mathbf{x}_k = [\mathbf{x}_{\boldsymbol{\eta}^1,k}^T \vdots \ldots \vdots \mathbf{x}_{\boldsymbol{\eta}^{N_p},k}^T \vdots \mathbf{x}_{\boldsymbol{\omega},k}^T]^T$ where $\mathbf{x}_{\boldsymbol{\eta}^z,k} = [x_{\eta_1^z,k} \ \ldots \ x_{\eta_P^z,k}]^T$ and $\mathbf{x}_{\boldsymbol{\omega},k} = [x_{\omega_1,k} \ \ldots \ x_{\omega_{N_n},k}]^T$ with $x_{q,k}$ denoting the modulated symbol supposed to be transmitted through the qth subchannel. Then, at the receiver, the $SL \times 1$ received symbol vector $\mathbf{y}_k = [\mathbf{y}_{\boldsymbol{\eta}^1,k}^T \vdots \ldots \vdots \mathbf{y}_{\boldsymbol{\eta}^{N_p},k}^T \vdots \mathbf{y}_{\boldsymbol{\omega},k}^T]^T$, where $\mathbf{y}_{\boldsymbol{\eta}^z,k} = [y_{\eta_1^z,k} \ \ldots \ y_{\eta_P^z,k}]^T$ and $\mathbf{y}_{\boldsymbol{\omega},k} = [y_{\omega_1,k} \ \ldots \ y_{\omega_{N_n},k}]^T$ with $y_{q,k}$ denoting the received symbol of the qth subchannel, is

$$\mathbf{y}_k = \breve{\mathbf{\Lambda}} \breve{\mathbf{\Theta}} \mathbf{x}_k + \mathbf{n}_k, \tag{5}$$

where $\breve{\mathbf{\Lambda}} = \text{diag}[\breve{\mathbf{\Lambda}}_{\boldsymbol{\eta}^1} \vdots \ldots \vdots \breve{\mathbf{\Lambda}}_{\boldsymbol{\eta}^{N_p}} \vdots \breve{\mathbf{\Lambda}}_{\boldsymbol{\omega}}]$ is a block diagonal matrix with entries $\breve{\mathbf{\Lambda}}_{\boldsymbol{\eta}^z} = \text{diag}[\lambda_{\eta_1^z} \ \ldots \ \lambda_{\eta_P^z}]$ and $\breve{\mathbf{\Lambda}}_{\boldsymbol{\omega}} = \text{diag}[\lambda_{\omega_1} \ \ldots \ \lambda_{\omega_{N_n}}]$, $\breve{\mathbf{\Theta}} = \text{diag}[\mathbf{\Theta} \vdots \ldots \vdots \mathbf{\Theta} \vdots \mathbf{I}_{N_n}]$ is a block diagonal matrix with $\mathbf{I}_{N_n}$ denoting the N_n-dimensional identity matrix, and $\mathbf{n}_k = [\mathbf{n}_{\boldsymbol{\eta}^1,k}^T \vdots \ldots \vdots \mathbf{n}_{\boldsymbol{\eta}^{N_p},k}^T \vdots \mathbf{n}_{\boldsymbol{\omega},k}^T]^T$ with $\mathbf{n}_{\boldsymbol{\eta}^z,k} = [n_{\eta_1^z,k} \ \ldots \ n_{\eta_P^z,k}]^T$ and $\mathbf{n}_{\boldsymbol{\omega},k} = [n_{\omega_1,k} \ \ldots \ n_{\omega_{N_n},k}]^T$ is an $SL \times 1$ noise vector with $n_{q,k}$ denoting the additive white Gaussian noise with zero mean and variance N_t/γ at the qth subchannel. Then, (5) is decomposed into $N_p + 1$ parts as

$$\begin{aligned} \mathbf{y}_{\boldsymbol{\eta}^z,k} &= \breve{\mathbf{\Lambda}}_{\boldsymbol{\eta}^z} \mathbf{\Theta} \mathbf{x}_{\boldsymbol{\eta}^z,k} + \mathbf{n}_{\boldsymbol{\eta}^z,k}, \\ \mathbf{y}_{\boldsymbol{\omega},k} &= \breve{\mathbf{\Lambda}}_{\boldsymbol{\omega}} \mathbf{x}_{\boldsymbol{\omega},k} + \mathbf{n}_{\boldsymbol{\omega},k}. \end{aligned} \tag{6}$$

In a similar way to BICMB-OFDM introduced in Section II, the location of the coded bit $c_{k'}$ within $\mathbf{x}_k$ is denoted as $k' \to (k, q, j)$, which means that $c_{k'}$ is mapped onto the jth bit position on the label of $x_{q,k}$. Then, the receiver calculates the ML bit metrics for $c_{k'} = b$ as

$$\Delta\left[\mathbf{y}_k, c_{k'}\right] = \begin{cases} \min\limits_{\mathbf{x} \in \psi_{c_{k'}}^{\hat{q},j}} ||\mathbf{y}_{\boldsymbol{\eta}^z,k} - \breve{\mathbf{\Lambda}}_{\eta^z} \mathbf{\Theta} \mathbf{x}||^2, & \text{if } q \in \boldsymbol{\eta}^z, \\ \min\limits_{x \in \chi_{c_{k'}}^j} \left| y_{q,k} - \lambda_q x \right|^2, & \text{if } q \in \boldsymbol{\omega}, \end{cases} \tag{7}$$

where $\hat{q} \in \{1, \ldots, P\}$ is the associated index of the qth subchannel in $\boldsymbol{\eta}^z$, and $\psi_b^{\hat{q},j}$ is a subset of χ^P defined as

$$\psi_b^{\hat{q},j} = \{\mathbf{x} = [x_1 \ \ldots \ x_P]^T : x_{p=\hat{q}} \in \chi_b^j, \text{ and } x_{p \neq \hat{q}} \in \chi\}.$$

Finally, the Viterbi decoding makes decisions as

$$\hat{\mathbf{c}} = \arg\min_{\mathbf{c}} \sum_{k'} \Delta\left[\mathbf{y}_k, c_{k'}\right]. \tag{8}$$

IV. Diversity Analysis

Equation (7) is similar to the bit metrics of precoded BICMB for flat fading MIMO channels presented in [8]. Hence, the average PEP between the transmitted codeword $\mathbf{c}$ and the decoded codeword $\hat{\mathbf{c}}$ can be upper bounded as

$$\Pr\left(\mathbf{c} \to \hat{\mathbf{c}}\right) \le \mathrm{E}\left[\frac{1}{2} \exp\left(-\frac{\sum_{k',d_H} \|\breve{\mathbf{\Lambda}} \breve{\mathbf{\Theta}} (\mathbf{x}_k - \hat{\mathbf{x}}_k)\|^2}{4N_0}\right)\right]. \tag{9}$$

where d_H is the Hamming distance between $\mathbf{c}$ and $\hat{\mathbf{c}}$, $\sum_{k',d_H}$ denotes the summation of the d_H values related to the different coded bits between the bit codewords, and $\hat{\mathbf{x}}_k$ is

$$\hat{\mathbf{x}}_k = \arg\min_{\mathbf{x} \in \xi_{\bar{c}_{k'}}^{q,j}} \|\mathbf{y}_k - \breve{\mathbf{\Lambda}} \breve{\mathbf{\Theta}} \mathbf{x}\|^2, \tag{10}$$

where $\xi_{\bar{c}_{k'}}^{q,j}$ is a subset of χ^{SL} defined as

$$\xi_b^{q,j} = \{\mathbf{x} = [x_1 \ \ldots \ x_{SL}]^T : x_{u=q} \in \chi_b^j, \text{ and } x_{u \neq q} \in \chi\}.$$

with $\bar{c}_{k'}$ denoting the complement of $c_{k'}$ in binary.

According to (6), the negative numerator of the exponent in (9) is rewritten as

$$\kappa = \sum_{k',d_H} \|\breve{\mathbf{\Lambda}}\breve{\mathbf{\Theta}}(\mathbf{x}_k - \hat{\mathbf{x}}_k)\|^2$$
$$= \sum_{z=1}^{N_p}\sum_{p=1}^{P} \lambda^2_{\eta^z_p} \sum_{k',d_{H,\boldsymbol{\eta}^z}} |\boldsymbol{\theta}_p^T\left(\mathbf{x}_{\boldsymbol{\eta}^z,k} - \hat{\mathbf{x}}_{\boldsymbol{\eta}^z,k}\right)|^2 +$$
$$\sum_{u=1}^{N_n} \lambda^2_{\omega_u} \sum_{k',d_{H,\omega_u}} |\left(x_{\omega_u,k} - \hat{x}_{\omega_u,k}\right)|^2, \tag{11}$$

where $\sum_{k',d_{H,\boldsymbol{\eta}^z}}$ and $\sum_{k',d_{H,\omega_u}}$ stand for the summation over the $d_{H,\boldsymbol{\eta}^z}$ and d_{H,ω_u} bit metrics related to the different coded bits carried on the subchannels in $\boldsymbol{\eta}^z$ and subchannel ω_u, respectively, and $\boldsymbol{\theta}_p^T$ denotes the pth row of $\mathbf{\Theta}$. By reordering the indices of singular values, (11) can be written as

$$\kappa = \sum_{q=1}^{SL} \rho_q \lambda_q^2, \tag{12}$$

with

$$\rho_q = \begin{cases} \sum\limits_{k',d_{H,\boldsymbol{\eta}^z}} |\boldsymbol{\theta}_{\hat{q}}^T\left(\mathbf{x}_{\boldsymbol{\eta}^z,k} - \hat{\mathbf{x}}_{\boldsymbol{\eta}^z,k}\right)|^2, & \text{if } q \in \boldsymbol{\eta}_z, \\ \sum\limits_{k',d_{H,\omega_u}} |\left(x_{\omega_u,k} - \hat{x}_{\omega_u,k}\right)|^2, & \text{if } q = \omega_u. \end{cases} \tag{13}$$

where $\hat{q}$ denotes the corresponding index of the qth subchannel in its precoded set. For BICMB-OFDM-SG, the subcarriers of each group are uncorrelated or weakly correlated [12], [13]. In that case, the matrices $\mathbf{\Lambda}(m)$ can be considered independent for each subcarrier group [12], [13]. Therefore, by converting the one-dimensional subchannel indices back to their corresponding two-dimensional indices, (9) is further rewritten as

$$\Pr\left(\mathbf{c} \to \hat{\mathbf{c}}\right) \le \prod_l \mathrm{E}\left[\exp\left(-\frac{\sum_s \rho_{l,s}\lambda_{l,s}^2}{4N_0}\right)\right]. \tag{14}$$

For each subcarrier, the terms inside the expectation in (14) can be upper bounded by employing the theorem proved in [8]. Hence, an upper bound of PEP is

$$\Pr\left(\mathbf{c} \to \hat{\mathbf{c}}\right) \le \prod_{l,\boldsymbol{\rho}_l \neq \mathbf{0}} \zeta_l \left(\frac{\rho_{l,min}}{4N_t}\gamma\right)^{-D_l}, \tag{15}$$

with $D_l = (N_r - \delta_l + 1)(N_t - \delta_l + 1)$, where $\rho_{l,min}$ denotes the minimum non-zero element in $\boldsymbol{\rho}_l$ whose element $\rho_{l,s}$ denotes the weight for $\lambda_{l,s}^2$, δ_l denotes the index of the first non-zero element in $\boldsymbol{\rho}_l$, and ζ_l is a constant. Therefore, the diversity is

$$D = \sum_{l,\boldsymbol{\rho}_l \neq \mathbf{0}} D_l. \tag{16}$$

Full diversity $N_r N_t L$ [1] can be achieved if and only if $\rho_{l,1} \neq 0, \forall l$ for all error events. Note that the full diversity condition is related to the combination of the precoding matrix, the convolutional code, and the bit interleaver.

V. Full-Diversity Precoding Design

The precoding design satisfying the full diversity condition $\rho_{l,1} \neq 0, \forall l$ of all error events may not be unique. In this section, a sufficient precoding design is developed for BICMB-OFDM-SG which guarantees full diversity while minimizing the increased decoding complexity caused by precoding. More discussion can be found in [15].

A. Choice of Precoding Matrix

An upper bound of PEP for BICMB-OFDM-SG in [12], [13] is similar to (14) only with different weights of

$$\tilde{\rho}_{l,s} = d_{min}^2 \alpha_{l,s}, \tag{17}$$

where d_{min} is the minimum Euclidean distance in the constellation, and $\alpha_{l,s}$ denotes the number of distinct bits transmitting through the sth subchannel of the lth subcarrier for an error path, which implies $\sum_{l=1}^{L}\sum_{s=1}^{S} \alpha_{l,s} = d_H$. The diversity can be derived in a similar fashion to (15) and (16), and the full diversity condition is $\alpha_{l,1} \neq 0, \forall l$ for all error events. As proved in [12], [13], the full diversity condition can be achieved if and only if $R_c SL \le 1$. The reason is that, if $R_c SL > 1$, there always exists at least one error path with no error bit transmitted through the first subchannel of a subcarrier.

It is obvious that $\rho_{l,1} = 0$ when $\alpha_{l,1} = 0$, if the $\{l, 1\}$th subchannel is non-precoded. However, if it is precoded, then $\rho_{l,1}$ could be nonzero even when $\alpha_{l,1} = 0$, depending on $\boldsymbol{\theta}_{\hat{q}}^T$ and each error event as shown in (13). Therefore, by proper precoding design, BICMB-OFDM-SG could achieve full diversity even if $R_c SL > 1$. Note that it is inconvenient to consider all error events which could be very large in number. However, since an error event only affects $\mathbf{x}_{\boldsymbol{\eta}^z,k} - \hat{\mathbf{x}}_{\boldsymbol{\eta}^z,k}$ in (13), a sufficient condition of the precoding design is

$$|\boldsymbol{\theta}_{\hat{q}}^T\left(\mathbf{x} - \hat{\mathbf{x}}\right)|^2 \neq 0, \text{ for } (q \bmod S) = 1, \tag{18}$$

of all pairs of $\mathbf{x}$ and $\hat{\mathbf{x}}$. It is not hard to find $\boldsymbol{\theta}_{\hat{q}}^T$ which satisfies (18). In fact, if each element in $\boldsymbol{\theta}_{\hat{q}}^T$ is nonzero, (18) is satisfied.

Note that the condition (18) is designed for certain rows of $\mathbf{\Theta}$ related to the first subchannel of each subcarrier. Although other subchannels do not affect the diversity as shown in (15) and (16), the condition (18) can be further simplified to

$$|\boldsymbol{\theta}_p^T\left(\mathbf{x} - \hat{\mathbf{x}}\right)|^2 \neq 0, \forall p, \tag{19}$$

of all different $\mathbf{x}$ and $\hat{\mathbf{x}}$.

Assume that the average transmitted power at each transmit antenna is the same, then the precoding matrix is chosen as

$$\theta_{u,v} \neq 0, \forall u, \forall v \text{ and } \|\boldsymbol{\theta}_p^T\|^2 = 1, \forall p. \tag{20}$$

In fact, the precoding matrices in [7] all satisfy the condition (20), which are considered in the following subsections.

B. Minimum Effective Dimension of Precoding Matrix

When the precoding matrices in [7] are applied, the weights of (13) can be simplified to

$$\rho_q = d_{min}^2 \beta_q, \tag{21}$$

where

$$\beta_q = \begin{cases} |\theta_{\hat{q},min}|^2 \alpha_{\boldsymbol{\eta}^z,min}, & \text{if } q \in \boldsymbol{\eta}_z, \\ \alpha_q, & \text{if } q = \omega_u, \end{cases} \quad (22)$$

with $\theta_{\hat{q},min}$ denoting the element in $\boldsymbol{\theta}_{\hat{q}}$ having the smallest absolute value and $\alpha_{\boldsymbol{\eta}^z,min}$ denoting the minimum nonzero α element related to the P precoded subchannels for the zth set. Since (21) is a lower bound for (13), an upper bound with a form similar to (15) and (16) can be derived with the simplified weights (21). Compared to the weights of (17) for non-precoded BICMB-OFDM-SG, the weights of the N_n non-precoded subchannels are the same. For the N_pP precoded subchannels, each weight now depends on the α elements of the P precoded subchannels of the corresponding set instead of only one subchannel. Therefore, if an errored bit is transmitted through a precoded subchannel, all weights in (21) for the P precoded subchannels of the corresponding set is nonzero. However, if no errored bit is transmitted through a precoded subchannel set, then all weights of the P precoded subchannels are zero, which are the same as non-precoded BICMB-OFDM-SG. If that happens, precoding is meaningless since the PEPs with the worst diversity dominate the overall performance. Therefore, the precoding design requirement is that at least an errored bit is carried on each precoded subchannel set.

The aforementioned precoding design requirement is related to the convolutional code, the bit interleaver, and the dimension of the precoding matrix. In fact, if $P = SL$, which means all subchannels are precoded by only one $SL \times SL$ precoding matrix $\boldsymbol{\Theta}$, the requirement can be easily satisfied. However, a larger dimension for $\boldsymbol{\Theta}$ results in higher complexity for calculating the metrics associated with the precoded bits in (7). Hence, the minimum effective dimension of $\boldsymbol{\Theta}$ should be found. Assume that $N_b = R_cSLJ$ information bits are transmitted, then J coded bits are transmitted by each of the SL parallel subchannels. Then, PJ coded bits are transmitted by a precoded subchannel set. Note that N_b information bits can provide 2^{N_b} different bit codewords. Hence, if PJ is smaller than N_b, there always exist at least a pair of bit codewords whose PJ coded bits transmitted by a precoded subchannel set are the same. The reason is that the total number of possible bit sequences of a precoded subchannel set 2^{PJ} is smaller than the total number of possible bit codewords of 2^{N_b}. As a result, the precoded subchannel set is non-effective. Therefore, PJ cannot be smaller than N_b, i.e., $P \geq R_cSL$. Since P is an integer, the minimum effective dimension of $\boldsymbol{\Theta}$ is $P = \lceil R_cSL \rceil$. Note that $P = \lceil R_cSL \rceil$ is only proved in this subsection to be a necessary condition because the requirement, i.e., at least an errored bit is transmitted through each precoded subchannel set, is also related to the convolutional code and the bit interleaver.

C. Minimum Effective Number of Precoding Subchannel Sets

Assume that at least an errored bit of each error event is carried on each precoded subchannel by a properly designed combination of the convolutional code and the bit interleaver, then every precoded subchannel set is effective. However, it still does not guarantee full diversity. Note that the full diversity condition requires that $\rho_{l,1} \neq 0, \forall l$ of all error events. It is presented in Section V-B that the non-precoded subchannel results in the same weight for both precoded and non-precoded BICMB-OFDM-SG. Hence, if a first subchannel of a subcarrier is not precoded, there always exists at least one error path with no errored bits carried on that subchannel if $R_cSL > 1$, as proved in [12], [13]. In that case, full diversity cannot be achieved even if all precoded subchannel sets are effective. Therefore, the first subchannels of all subcarriers should be precoded. Since there are L subcarriers, and each $\boldsymbol{\Theta}$ can precode $P \geq \lceil R_cSL \rceil$ subchannels, the minimum effective number of $\boldsymbol{\Theta}$ is $N_p = \lceil L/P \rceil$.

Note that if $\lceil L/P \rceil P > SL$, not all first subchannels of all subcarriers can be precoded by $\boldsymbol{\Theta}$ with effective dimension $P \geq \lceil R_cSL \rceil$. In fact, the case of $\lceil L/P \rceil P > SL$ can only happen when $S = 1$. In other words, when $S \geq 2$, $\lceil L/P \rceil P \leq SL$ always valid, which is proved below.

Proof: Note that $\lceil L/P \rceil \geq 1$. If $\lceil L/P \rceil = 1$, $\lceil L/P \rceil P \leq SL$ always valid because $P \leq SL$. On the other hand, if $\lceil L/P \rceil \geq 2$, then $P < L$. Because $S \geq 2$, then

$$\lceil L/P \rceil P < 2L \leq SL. \quad (23)$$

This concludes the proof. □

Therefore, if $S \geq 2$, then $N_p = \lceil L/P \rceil$ with $P = \lceil R_cSL \rceil$. Similar selection of N_p and P can be applied for $S = 1$ if $N_pP \leq L$. Otherwise, if $N_pP > L$ for $S = 1$, the dimension P of $\boldsymbol{\Theta}$ needs to be increased until $N_pP \leq L$ is satisfied.

Similarly, the minimum effective selection of P and N_p is only a necessary condition to achieve full diversity, because the convolutional code and the interleaver also need to be considered to satisfy the requirement, i.e., at least one errored bit is transmitted through each precoded subchannel set.

D. Selection of Precoded Subchannels

Based on (15), (16), (21), and (22), the diversity of BICMB-OFDM-SG with precoding also depends on the α-spectra of non-precoded BICMB-OFDM-SG, which are related to the bit interleaver and the trellis of the convolutional code, and are independent of $\boldsymbol{\Theta}$. The α-spectra can be derived by a similar approach to BICMB of flat fading MIMO channels presented in [5], or by computer search. A method is provided in [13] to derive the α-spectra. Based on the α-spectra, the selection of precoded subchannels should be properly designed to satisfy the condition of $\rho_{l,1} \neq 0, \forall l$ for all error events. In this section, the minimum effective selection of P and N_p has been derived as a necessary condition to offer full diversity. In the following, it is proved sufficient to provide full diversity with the joint design of the precoded subchannels and bit interleaver.

Consider a convolutional code with rate $R_c = k_c/n_c$. If the spatial de-multiplexer is not a random switch, the period of the spatial de-multiplexer is an integer multiple of the Least Common Multiple (LCM) of n_c and SL. Note that a period of the bit interleaver is restricted to correspond to an integer multiple of trellis sections. Define $Q = LCM(n_c, SL)$ as the number of coded bits for a minimum period. Since each subchannel needs to be evenly employed for a period, $Q/(SL)$ coded bits are assigned on each subchannel. Therefore, $QP/(SL)$ coded bits are carried on one precoded subchannel set, which offer the same effect on the diversity. To guarantee $\rho_{l,1} \neq 0$, $\forall l$ of all error events, it is sufficient to consider only the first branches that split from the zero state in one period because of the repetition property of the convolutional code. In other words, if at least one coded bit for each precoded subchannel set is assigned for each branch, full diversity is achieved. Note that there are QR_c branches in a period. Since $P \geq R_cSL$, then $QP/(SL) \geq QR_c$. Hence, all branches in a period can be assigned at least one coded bit carried on each precoded subchannel set, which guarantees full diversity.

E. Full-Diversity Precoding Design Summary

Based on the discussion in this section, a sufficient method of the full-diversity precoding design for BICMB-OFDM-SG with $R_cSL > 1$ is summarized as the following steps.

1) Calculate $P = \lceil R_cSL \rceil$.
2) Calculate $N_p = \lceil L/P \rceil$.
3) Calculate N_pP. If $N_pP > SL$, set $P = P + 1$ and go to 2). Otherwise, go to 4).
4) Select N_pP precoded subchannels which include all the L first subchannels of all subcarriers.
5) Design a bit interleaver pattern of $Q = LCM(n_c, SL)$ coded bits by assigning one precoded subchannel from each set to each branch for a period.

VI. Simulation Results

To verify the proposed full-diversity precoding design, 2×2, $M = 64$ BICMB-OFDM-SG with $L = 2$ and $L = 4$ using 4-QAM are considered for simulations. The number of employed subchannels for each subcarrier and the dimension of precoding matrix for each precoded subchannel set are assumed to be the same, respectively. The generator polynomials in octal for the convolutional codes with $R_c = 1/4$ and $R_c = 1/2$ are $(5,7,7,7)$, and $(5,7)$ respectively, and the codes with $R_c = 2/3$ and $R_c = 4/5$ are punctured from the $R_c = 1/2$ code. Each OFDM symbol has 4μs duration, of which 0.8μs is CP, and $L_{cp} = 16$. Equal power channel taps are considered. The bit interleaver employs simple rotation for BICMB-OFDM-SG with non-effective precoding selections and without precoding. For BICMB-OFDM-SG with effective precoding selections, the proposed full-diversity precoding design is employed. In the figures, NP denotes non-precoded. Unequal power channel taps are not considered since they do not affect the maximum achievable diversity as discussed in [12]. Note that simulations of 2×2, $L = 2$ and $L = 4$ BICMB-OFDM-SG are shown because the diversity values could be investigated explicitly through figures. More results can be found in [15]. In our simulations, we will show that the maximum diversity of N_rN_tL [1] is achieved.

Fig. 3 shows the Bit Error Rate (BER) performance of 2×2, $L = 2$, $M = 64$, $S = 2$ BICMB-OFDM-SG with and without precoding for different R_c. For $R_c = 1/4$, full diversity of 8 can be achieved even without precoding since $R_cSL \leq 1$ [12], [13]. However, in the cases of $R_c = 1/2$, $R_c = 2/3$, and $R_c = 4/5$, the diversity orders are 5, 2, and 1 respectively, and the full diversity degradations result from $R_cSL > 1$. On the other hand, full diversity can be restored by employing the proposed full-diversity precoding design. The selections are $\{P = 2, N_p = 1\}$, $\{P = 3, N_p = 1\}$, and $\{P = 4, N_p = 1\}$ for $R_c = 1/2$, $R_c = 2/3$, and $R_c = 4/5$, respectively.

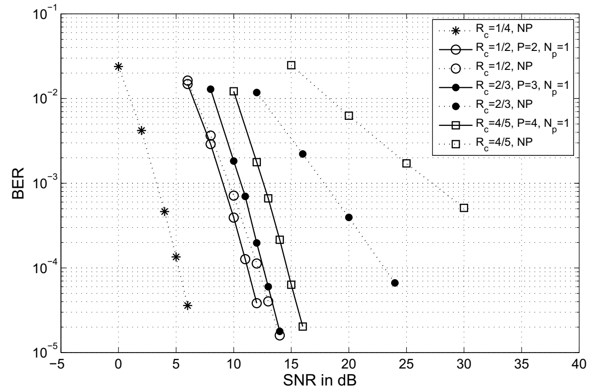

Fig. 3. BER vs. SNR for 2×2, $L = 2$, $M = 64$, $S = 2$ BICMB-OFDM-SG with and without precoding.

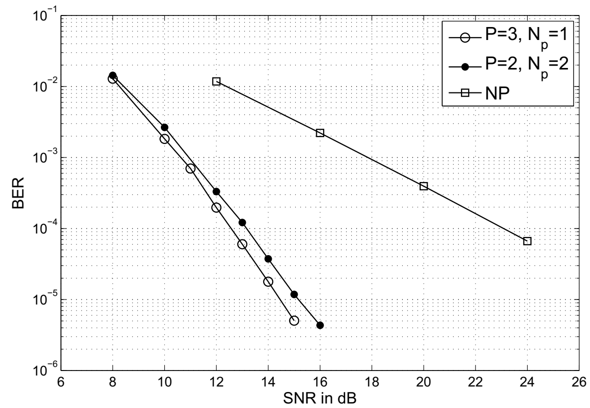

Fig. 4. BER vs. SNR for 2×2, $L = 2$, $M = 64$, $S = 2$, $R_c = 2/3$ BICMB-OFDM-SG with different precoding selections and without precoding.

Fig. 4 shows the BER performance of 2×2, $L = 2$, $M = 64$, $S = 2$, $R_c = 2/3$ BICMB-OFDM-SG with precoding for different selections of P and N_p and without precoding. Since $R_cSL > 1$, full diversity cannot be achieved without precoding, and the diversity is 2. On the other hand, the efficient selection $\{P = 3, N_p = 1\}$ can offer full diversity, while $\{P = 2, N_p = 2\}$ with diversity of 5 cannot provide full diversity because $\lceil R_cSL \rceil = 3$. Note that for $\{P = 2, N_p = 2\}$, to offer relatively high diversity, the first subchannel of the first subcarrier is precoded with the second subchannel of the second subcarrier, while the other two subchannels are precoded.

Fig. 5 shows the BER performance of 2×2, $L = 4$, $M = 64$, $S = 1$ BICMB-OFDM-SG with and without precoding for

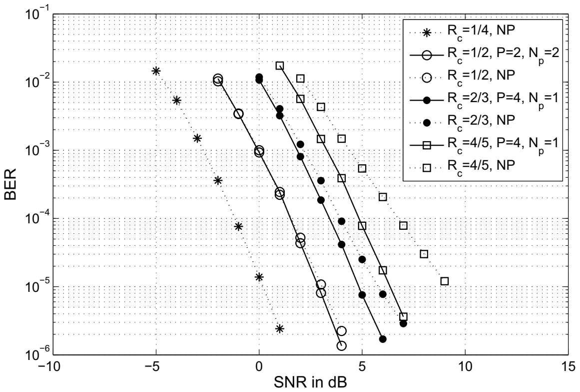

Fig. 5. BER vs. SNR for 2×2, $L = 4$, $M = 64$, $S = 1$ BICMB-OFDM-SG with and without precoding.

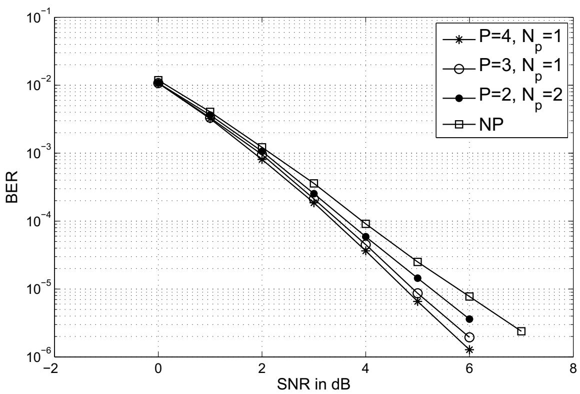

Fig. 6. BER vs. SNR for 2×2, $L = 4$, $M = 64$, $S = 1$, $R_c = 2/3$ BICMB-OFDM-SG with different precoding selections and without precoding.

different R_c. For $R_c = 1/4$, full diversity of 16 can be offered even without precoding because $R_cSL \leq 1$ [12], [13]. However, for $R_c = 1/2$, $R_c = 2/3$, and $R_c = 4/5$, the diversity orders are 12, 8, and 4 respectively, and full diversity orders are not achieved since $R_cSL > 1$. On the other hand, full diversity can be restored by the proposed full-diversity precoding design. The selections are $\{P = 2, N_p = 2\}$, $\{P = 4, N_p = 1\}$, and $\{P = 4, N_p = 1\}$ for $R_c = 1/2$, $R_c = 2/3$, and $R_c = 4/5$, respectively.

Fig. 6 shows the BER performance of 2×2, $L = 4$, $M = 64$, $S = 1$, $R_c = 2/3$ BICMB-OFDM-SG with precoding for different selections of P and N_p and without precoding. Because $R_cSL > 1$, full diversity cannot be achieved without precoding, and the diversity is 8. On the other hand, $P = 4$ $N_p = 1$ is an effective precoding selection to provide the full diversity of 16, while $\{P = 3, N_p = 1\}$ and $\{P = 2, N_p = 2\}$ with diversity orders of 12 and 8 respectively cannot offer full diversity because $\lceil L/P \rceil P > L$ and $\lceil R_cSL \rceil = 3$ respectively. Note that to achieve relatively high diversity, for $P = 3$ $N_p = 1$, the subchannel of the second subcarrier is non-precoded, while for $\{P = 2, N_p = 2\}$, the subchannel of the first subcarrier is precoded with the subchannel of the third subcarrier, and the other two subchannels are precoded.

The results show that for BICMB-OFDM-SG without the condition $R_cSL \leq 1$, full diversity can be provided by the proposed full-diversity precoding design proposed. The performance improvement is significant if the diversity without the proposed design is relatively small compared to full diversity. On the other hand, this advantage may start at the SNR offering very low BER if the diversity without the design is close to full diversity. In that case, its value depends on the BERs of different applications.

VII. Conclusions

In this paper, a full-diversity precoding design is developed for BICMB-OFDM without the full diversity restriction of $R_cSL \leq 1$. The design provides a sufficient method to guarantee full diversity while minimizing the increased decoding complexity caused by precoding. With this method, more choices are offered with different trade-offs among performance, transmission rate, and decoding complexity. As a result, BICMB-OFDM becomes a more flexible broadband wireless communication technique.

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