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Constellation Precoded Multiple Beamforming

Hong Ju Park, Boyu Li, *Student Member, IEEE*, and Ender Ayanoglu, *Fellow, IEEE*

Abstract—Beamforming techniques that employ Singular Value Decomposition (SVD) are commonly used in Multi-Input Multi-Output (MIMO) wireless communication systems. In the absence of channel coding, when a single symbol is transmitted, these systems achieve the full diversity order provided by the channel; whereas when multiple symbols are simultaneously transmitted, this property is lost. When channel coding is employed, full diversity order can be achieved. For example, when Bit-Interleaved Coded Modulation (BICM) is combined with this technique, full diversity order of NM in an $M \times N$ MIMO channel transmitting S parallel streams is possible, provided a condition on S and the BICM convolutional code rate is satisfied. In this paper, we present constellation precoded multiple beamforming which can achieve the full diversity order both with BICM-coded and uncoded SVD systems. We provide an analytical proof of this property. To reduce the computational complexity of Maximum Likelihood (ML) decoding in this system, we employ Sphere Decoding (SD). We report an SD technique that reduces the computational complexity beyond commonly used approaches to SD. This technique achieves several orders of magnitude reduction in computational complexity not only with respect to conventional ML decoding but also, with respect to conventional SD.

Index Terms—MIMO systems, SVD, BICMB, constellation precoding, sphere decoding.

I. INTRODUCTION

WHEN the perfect channel state information is available at the transmitter, beamforming is employed to achieve spatial multiplexing and thereby increase the data rate, or to enhance the performance of a Multiple-Input Multiple-Output (MIMO) system [1]. The beamforming vectors are designed in [2], [3] for various design criteria, and can be obtained by the Singular Value Decomposition (SVD), leading to a channel-diagonalizing structure optimum in minimizing the average Bit Error Rate (BER) [3]. Uncoded Single Beamforming (SB), which carries only one symbol at a time, was shown to

achieve the full diversity order of NM where N and M are the number of transmit and receive antennas, respectively [4], [5]. However, the diversity order of uncoded multiple beamforming, which increases the throughput by sending multiple symbols at a time, is $(N - S + 1)(M - S + 1)$ where the symbols are transmitted on the subchannels with the largest S singular values, losing the full diversity order over flat fading channel [4], [5].

It is known that an SVD subchannel with larger singular value provides larger diversity gain [5]. Under the simultaneous parallel transmission of the symbols on the diagonalized subchannels, the performance at high Signal-to-Noise Ratio (SNR) is dominated by the subchannel with the smallest singular value. To overcome the degradation of the diversity order of multiple beamforming, Bit-Interleaved Coded Multiple Beamforming (BICMB) was proposed [6], [7]. This scheme interleaves the codewords through the multiple subchannels with different singular value, resulting in better diversity order. BICMB can achieve the full diversity order offered by the channel as long as the code rate R_c and the number of employed subchannels S satisfy the condition $R_c S \leq 1$ [8].

In this paper, we present a multiple beamforming technique that achieves the full diversity order in both coded and uncoded systems. This technique employs the constellation precoding scheme [9], [10], [11], [12], [13], which is used for space-time or space-frequency block codes to increase the system data rate without losing the full diversity order. We show via a Pairwise Error Probability (PEP) analysis that Fully Precoded Multiple Beamforming (FPMB) with Maximum Likelihood Decoding (MLD) achieves the full diversity order even in the absence of any channel coding. We also present the diversity analysis of Bit-Interleaved Coded Multiple Beamforming with Constellation Precoding (BICMB-CP), which adds the constellation precoding stage to BICMB. We show that the addition of the constellation precoder to BICMB, whose code rate R_c is larger than $1/S$, provides the full diversity when the subchannels for the precoded symbols are properly chosen. Simulation results are shown to confirm the analysis.

Multiple beamforming without constellation precoding separates the MIMO channel into independent parallel subchannels, enabling symbol-by-symbol detection on each subchannel. Since the precoder at the transmitter no longer allows for the parallel independent detection of the symbols on each subchannel, the complexity of MLD for precoded symbols, which provides optimal performance, increases exponentially

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H. J. Park was and B. Li and E. Ayanoglu are with the Center for Pervasive Communications and Computing, Department of Electrical Engineering and Computer Science, The Henry Samueli School of Engineering, University of California, Irvine, CA 92697-3975 USA (e-mail: {boyul, ayanoglu}@uci.edu).

H. J. Park is with Samsung Electronics, Suwon, Korea (e-mail: honspark@samsung.com).

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with the number of possible constellation points of the modulation scheme and the dimension of the constellation precoder. The complexity increase makes the receiver with MLD unsuitable for practical purposes [14]. On the other hand, Sphere Decoding (SD) was proposed as an alternative for MLD that provides optimal performance with reduced computational complexity [15].

Several complexity reduction techniques for SD have been proposed. In [16] and [17], attention is drawn to the initial radius selection strategy, since an inappropriate initial radius can result in either a large number of lattice points to be searched, or a number of restarted searches with increased initial radius. In [18] and [19], the complexity is reduced by making a proper choice to update the sphere radius. Other methods, such as the K -best lattice decoder [20], [21], and a combination of SD and K -best decoder [22], can significantly reduce the complexity of low SNR at the cost of BER performance.

In this paper, we propose an SD algorithm which efficiently improves the complexity of constellation precoded multiple beamforming over flat fading channels by reducing the average number of multiplications required to obtain the optimal solution. This complexity reduction is accomplished by precalculating the multiplications at the beginning of decoding, and recycling them later for the repetitive calculations. The reduction is achieved further by the help of the lattice representation of our previous work presented in [23], which introduces orthogonality between the real and imaginary parts of every detected symbol. Based on Zero-Forcing Decision Feedback Equalization (ZF-DFE), the proposed SD algorithm includes a method to determine the initial radius, reducing the average number of real multiplications needed to acquire one precoded bit metric for BICMB-CP. With simulation results, we show that conventional SD reduces the complexity substantially compared with the exhaustive search, and the complexity can be further reduced effectively by our proposed SD. The complexity reduction becomes larger as the constellation precoder dimension and the constellation size become larger.

The rest of this paper is organized as follows. The description of uncoded and coded multiple beamforming combined with constellation precoding is given in Section II. Sections III and IV present the diversity analysis of the MIMO schemes through the calculation of the upper bound to PEP. The computational complexity reduction sphere detection algorithm is discussed in Section V. Simulation results supporting the analysis are shown in Section VI. We discuss possibilities for a simplified decoder in Section VII. Finally, we end the paper with our conclusion in Section VIII.

Notation: Bold lower (upper) case letters denote vectors (matrices). $\text{diag}[\mathbf{B}_1, \dots, \mathbf{B}_P]$ stands for a block diagonal matrix with matrices $\mathbf{B}_1, \dots, \mathbf{B}_P$, and $\text{diag}[b_1, \dots, b_P]$ is a diagonal matrix with diagonal entries b_1, \dots, b_P . $\Re(\cdot)$ and $\Im(\cdot)$ denote the real and imaginary part of a complex number, respectively. The superscripts $(\cdot)^H$, $(\cdot)^T$, $(\cdot)^*$, (\cdot) stand for conjugate transpose, transpose, complex conjugate, binary complement, respectively, and \forall denotes for-all. $\lceil \cdot \rceil$ is the ceiling function that maps a real number to the next largest integer. \mathbb{R}^+ and \mathbb{C} stand for the set of positive real numbers

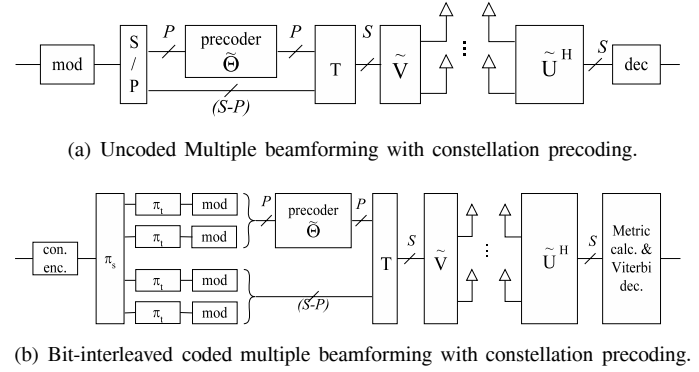


Fig. 1. Structure of constellation precoded multiple beamforming. Matrices $\tilde{\mathbf{U}}$ and $\tilde{\mathbf{V}}$ consist of the first S vectors of \mathbf{U} and \mathbf{V} in the SVD decomposition $\mathbf{H} = \mathbf{U}\mathbf{\Lambda}\mathbf{V}^H$.

and the complex numbers, respectively. d_{min} is the minimum Euclidean distance between two points in a constellation.

II. SYSTEM MODEL

A. Uncoded Multiple Beamforming with Constellation Precoding

Uncoded Multiple Beamforming with Constellation Precoding (UMB-CP) transforms modulated symbols to precoded symbols via a precoding matrix as depicted in Fig. 1(a). The $S \times 1$ symbol vector \mathbf{x} , where $S \leq \min(N, M)$, is precoded by a square matrix Θ . We assume that the elements of \mathbf{x} belong to a signal set $\chi \subset \mathbb{C}$ of size $|\chi| = 2^m$, such as 2^m -QAM, where m is the number of input bits to the Gray encoder. The permutation matrix \mathbf{T} reorders the precoded P symbols and non-precoded $S - P$ symbols to be transmitted on the predefined subchannels created by the SVD of the MIMO channel. Let us define $\boldsymbol{\eta} = [\eta_1 \dots \eta_P]$ as a vector whose element η_p is the index of the subchannel on which the precoded symbols are transmitted, and ordered increasingly such that $\eta_p < \eta_q$ for $p < q$. In the same way, $\boldsymbol{\omega} = [\omega_1 \dots \omega_{(S-P)}]$ is defined as an increasingly ordered vector whose elements are the indices of the subchannels which carry the non-precoded symbols.

The serial-to-parallel converter organizes the symbol vector \mathbf{x} as $\mathbf{x} = [\mathbf{x}_\eta^T; \mathbf{x}_\omega^T]^T = [x_{\eta_1} \dots x_{\eta_P}; x_{\omega_1} \dots x_{\omega_{(S-P)}}]^T$, where \mathbf{x}_η and \mathbf{x}_ω consist of the modulated entries to be transmitted on the subchannels specified in $\boldsymbol{\eta}$ and $\boldsymbol{\omega}$, respectively. The $S \times 1$ detected symbol vector $\mathbf{y} = [\mathbf{y}_p^T; \mathbf{y}_n^T]^T = [y_1 \dots y_P; y_{P+1} \dots y_S]^T$ at the receiver is written as

$$\mathbf{y} = \mathbf{\Gamma}\mathbf{\Theta}\mathbf{x} + \mathbf{n} \quad (1)$$

where $\mathbf{\Gamma}$ is a block diagonal matrix, $\mathbf{\Gamma} = \text{diag}[\mathbf{\Gamma}_p, \mathbf{\Gamma}_n]$, with diagonal matrices defined as $\mathbf{\Gamma}_p = \text{diag}[\lambda_{\eta_1}, \dots, \lambda_{\eta_P}]$, $\mathbf{\Gamma}_n = \text{diag}[\lambda_{\omega_1}, \dots, \lambda_{\omega_{(S-P)}}]$ and $\lambda_s \in \mathbb{R}^+$ is the s^{th} singular value of \mathbf{H} , in decreasing order. The vector $\mathbf{n} = [\mathbf{n}_p^T; \mathbf{n}_n^T]^T$ is additive white Gaussian noise with zero mean and variance $N_0 = N/\text{SNR}$. The matrix \mathbf{H} is complex Gaussian with zero mean and unit variance, and to make the received signal-to-noise ratio SNR , the total transmitted power is scaled as N .

The matrix Θ is a block diagonal matrix $\Theta = \text{diag}[\tilde{\Theta}, \mathbf{I}_{S-P}]$ and $\tilde{\Theta}$ is a $P \times P$ constellation precoding matrix that precodes the first P modulated symbols of the vector \mathbf{x} . Then, the input-output relation in (1) is decomposed into two equations as

$$\begin{aligned} \mathbf{y}_p &= \Gamma_p \tilde{\Theta} \mathbf{x}_\eta + \mathbf{n}_p \\ \mathbf{y}_n &= \Gamma_n \mathbf{x}_\omega + \mathbf{n}_n. \end{aligned} \quad (2)$$

When all of the S modulated symbols are precoded ($P = S$), we call the resulting system Fully Precoded Multiple Beamforming (FPMB), otherwise, we call it Partially Precoded Multiple Beamforming (PPMB). Partial precoding can result in reduced complexity and therefore can be desirable. As will be illustrated in the sequel, uncoded PPMB does not achieve the full diversity order provided by the MIMO channel, but when combined with BICM (BICMB-PP), it can achieve this performance with less complexity than FPMB combined with BICM (BICMB-FP).

MLD of the detected symbol $\hat{\mathbf{x}} = [\hat{\mathbf{x}}_\eta^T : \hat{\mathbf{x}}_\omega^T]^T = [\hat{x}_{\eta_1} \cdots \hat{x}_{\eta_P} : \hat{x}_{\omega_1} \cdots \hat{x}_{\omega_{(S-P)}}]^T$ is given by

$$\hat{\mathbf{x}} = \arg \min_{\mathbf{x} \in \chi^S} \|\mathbf{y} - \Gamma \Theta \mathbf{x}\|^2 \quad (3)$$

where χ^S represents the S -dimensional product space based on χ . For PPMB, the symbol can be detected in a parallel fashion as

$$\hat{\mathbf{x}}_\eta = \arg \min_{\mathbf{x} \in \chi^P} \|\mathbf{y}_p - \Gamma_p \tilde{\Theta} \mathbf{x}\|^2 \quad (4)$$

for the precoded symbol, and

$$\hat{x}_l = \arg \min_{x \in \chi} |y_l - \lambda_{\tilde{l}} x|^2 \quad (5)$$

for the non-precoded symbol where \tilde{l} is the corresponding index transformed by \mathbf{T} .

B. Bit-Interleaved Coded Multiple Beamforming with Constellation Precoding

Fig. 1(b) represents the structure of Bit-Interleaved Coded Multiple Beamforming with Constellation Precoding (BICMB-CP). First, the convolutional encoder with code rate $R_c = k_c/n_c$, possibly combined with a perforation matrix for a high rate punctured code, generates the codeword \mathbf{c} from the information bits. Then, the spatial interleaver π_s distributes the coded bits into S streams, each of which is interleaved by an independent bit-wise interleaver π_t . The interleaved bits are mapped by Gray encoding onto the symbol sequence $\mathbf{X} = [\mathbf{x}_1 \cdots \mathbf{x}_K]$, where \mathbf{x}_k is an $S \times 1$ symbol vector at the k^{th} time instant. Each entry of \mathbf{x}_k belongs to a signal set χ .

The symbol vector \mathbf{x}_k is multiplied by the $S \times S$ precoder Θ . When all of the S modulated entries are precoded ($P = S$), we call the resulting system Bit-Interleaved Coded Multiple Beamforming with Full Precoding (BICMB-FP), otherwise, we call it Bit-Interleaved Coded Multiple Beamforming with Partial Precoding (BICMB-PP). The precoded symbol vector is transmitted on the MIMO channel described in Section II-A.

As in UMB-CP, the spatial interleaver arranges the symbol vector \mathbf{x}_k as $\mathbf{x}_k = [\mathbf{x}_{k,\eta}^T : \mathbf{x}_{k,\omega}^T]^T = [x_{k,\eta_1} \cdots x_{k,\eta_P} : x_{k,\omega_1} \cdots x_{k,\omega_{(S-P)}}]^T$. The $S \times 1$ detected symbol vector $\mathbf{r}_k = [r_{k,1} \cdots r_{k,P} : r_{k,P+1} \cdots r_{k,S}]^T$ at the k^{th} time instant is

$$\mathbf{r}_k = \Gamma \Theta \mathbf{x}_k + \mathbf{n}_k \quad (6)$$

where $\mathbf{n}_k = [(\mathbf{n}_k^p)^T : (\mathbf{n}_k^n)^T]^T$ is an additive white Gaussian noise vector.

The location of the coded bit $c_{k'}$ within the symbol sequence \mathbf{X} is known as $k' \rightarrow (k, l, i)$, where k , l , and i are the time instant in \mathbf{X} , the symbol position in \mathbf{x}_k , and the bit position on the label $x_{k,l}$, respectively. Let χ_b^i denote a subset of χ whose labels have $b \in \{0, 1\}$ in the i^{th} bit position. By using the location information and the input-output relation in (6), the receiver calculates the maximum likelihood bit metrics for the coded bit $c_{k'}$ as

$$\gamma^{l,i}(\mathbf{r}_k, c_{k'}) = \min_{\mathbf{x} \in \xi_{c_{k'}}^{l,i}} \|\mathbf{r}_k - \Gamma \Theta \mathbf{x}\|^2 \quad (7)$$

where $\xi_{c_{k'}}^{l,i}$ is a subset of χ^S , defined as

$$\xi_b^{l,i} = \{\mathbf{x} = [x_1 \cdots x_S]^T : x_{s|s=l} \in \chi_b^i, \text{ and } x_{s|s \neq l} \in \chi\}.$$

In particular, based on the decomposition of (6) similar to (4) and (5), the bit metrics, equivalent to (7) for partial precoding, are

$$\gamma^{l,i}(\mathbf{r}_k, c_{k'}) = \begin{cases} \min_{\mathbf{x} \in \psi_{c_{k'}}^{l,i}} \|\mathbf{r}_k^p - \Gamma_p \tilde{\Theta} \mathbf{x}\|^2, & \text{if } 1 \leq l \leq P \\ \min_{x \in \chi_{c_{k'}}^i} |r_{k,l} - \lambda_{\tilde{l}} x|^2, & \text{if } P+1 \leq l \leq S \end{cases} \quad (8)$$

where $\psi_b^{l,i}$ is a subset of χ^P , defined as

$$\psi_b^{l,i} = \{\mathbf{x} = [x_1 \cdots x_P]^T : x_{s|s=l} \in \chi_b^i, \text{ and } x_{s|s \neq l} \in \chi\},$$

and \tilde{l} is an entry in ω , corresponding to the subchannel mapped by \mathbf{T} . Finally, MLD makes decisions according to the rule

$$\hat{\mathbf{c}} = \arg \min_{\mathbf{c}} \sum_{k'} \gamma^{l,i}(\mathbf{r}_k, \tilde{c}_{k'}). \quad (9)$$

III. DIVERSITY ANALYSIS : UMB-CP

A. Fully Precoded Multiple Beamforming

Based on the MLD in (3), the upper bound to the instantaneous PEP between the transmitted symbol \mathbf{x} and the detected symbol $\hat{\mathbf{x}}$ is calculated as

$$\begin{aligned} \Pr(\mathbf{x} \rightarrow \hat{\mathbf{x}} | \mathbf{H}) &= \Pr(\|\mathbf{y} - \Gamma \Theta \mathbf{x}\|^2 \geq \|\mathbf{y} - \Gamma \Theta \hat{\mathbf{x}}\|^2 | \mathbf{H}) \\ &\leq \frac{1}{2} \exp\left(-\frac{\|\Gamma \Theta (\mathbf{x} - \hat{\mathbf{x}})\|^2}{4N_0}\right). \end{aligned} \quad (10)$$

Let $\mathbf{d} = [d_1 \cdots d_S]^T = \Theta(\mathbf{x} - \hat{\mathbf{x}})$. Then, for FPMB, the average PEP becomes

$$\Pr(\mathbf{x} \rightarrow \hat{\mathbf{x}}) \leq E \left[\frac{1}{2} \exp\left(-\frac{\sum_{s=1}^S \lambda_s^2 |d_s|^2}{4N_0}\right) \right]. \quad (11)$$

In [8], we showed that equations in the form of (11) have a closed form upper bound expression. We provide a formal statement below.

Theorem 1: Consider the $S \leq \min(N, M)$ ordered eigenvalues $\mu_1 > \dots > \mu_S$ of the uncorrelated central Wishart matrix¹ [24], and a weight vector $\phi = [\phi_1 \dots \phi_S]^T$ with non-negative real elements. In the high signal-to-noise ratio regime, an upper bound for the expression $E[\exp(-\gamma \sum_{s=1}^S \phi_s \mu_s)]$ which is used in the diversity analysis of a number of MIMO systems is

$$E \left[\exp \left(-\gamma \sum_{s=1}^S \phi_s \mu_s \right) \right] \leq \zeta (\phi_{\min} \gamma)^{-(N-\delta+1)(M-\delta+1)}$$

where γ is signal-to-noise ratio, ζ is a constant, $\phi_{\min} = \min_{\phi_i \neq 0} \{\phi_i\}_{i=1}^S$, and δ is the index indicating the first nonzero element in the weight vector.

Proof: See [8]. \square

Applying Theorem 1 to (11), we get the upper bound to PEP as

$$\Pr(\mathbf{x} \rightarrow \hat{\mathbf{x}}) \leq \tilde{\zeta} \left(\frac{\hat{d}_{\min}}{4N} \text{SNR} \right)^{-(N-\delta+1)(M-\delta+1)} \quad (12)$$

where $\tilde{\zeta}$ is a constant, $\hat{d}_{\min} = \min\{|d_1|^2, \dots, |d_S|^2\}$, and δ is an index indicating the first nonzero element of the vector $[|d_1|^2 \dots |d_S|^2]$. Therefore, FPMB achieves the full diversity order if δ from any distinct pair is equal to 1, which implies that $|d_1|^2 = |\theta_1^T(\mathbf{x} - \hat{\mathbf{x}})|^2 > 0$ for any distinct pair, where θ_1^T is the first row vector of Θ . Several methods to build the precoding matrix are described in [25] and [26].

B. Partially Precoded Multiple Beamforming

Generalizing (10) for PPMB, we get an upper bound to PEP as

$$\Pr(\mathbf{x} \rightarrow \hat{\mathbf{x}}) \leq E \left[\frac{1}{2} \exp \left(-\frac{\kappa}{4N_0} \right) \right] \quad (13)$$

where

$$\kappa = \sum_{s=1}^P \lambda_{\eta_s}^2 |\tilde{d}_s|^2 + \sum_{s=1}^{S-P} \lambda_{\omega_s}^2 |x_{\omega_s} - \hat{x}_{\omega_s}|^2 \quad (14)$$

and \tilde{d}_s is the s^{th} element of a vector $\tilde{\mathbf{d}} = \tilde{\Theta}(\mathbf{x}_{\eta} - \hat{\mathbf{x}}_{\eta})$. Let us assume that the constellation precoding matrix $\tilde{\Theta}$ meets the condition of FPMB to achieve the full diversity order. Since the expression (13) with (14) has a closed form expression similar to (12) as described in FPMB, the δ value needs to be obtained from a composite vector with the elements as $|\tilde{d}_s|^2$ and $|x_{\omega_s} - \hat{x}_{\omega_s}|^2$, to observe the diversity behavior of a given pairwise error. In addition, a different pair can lead to different diversity behavior. Therefore, we need to get the maximum δ out of all the possible pairwise errors to decide the diversity order of a given PPMB system.

All of the distinct pairs of \mathbf{x} and $\hat{\mathbf{x}}$ are divided into three groups in terms of \mathbf{x}_{η} , $\hat{\mathbf{x}}_{\eta}$, \mathbf{x}_{ω} , and $\hat{\mathbf{x}}_{\omega}$. The first group

¹A central Wishart matrix is the Hermitian matrix $\mathbf{A}\mathbf{A}^H$ where the entry of the matrix \mathbf{A} is complex Gaussian with zero mean so that $E[\mathbf{A}] = \mathbf{0}$. The Wishart matrix $\mathbf{A}\mathbf{A}^H$ is called uncorrelated if the common covariance matrix, defined as $\mathbf{C} = E[\mathbf{a}_s \mathbf{a}_s^H] \forall s$, where \mathbf{a}_s is the s^{th} column vector of \mathbf{A} , satisfies $\mathbf{C} = \mathbf{I}$.

TABLE I
DIVERSITY ORDER (O_{div}) OF 4×4 , $S = 4$ PARTIALLY PRECODED
MULTIPLE BEAMFORMING SYSTEM

P	η	ω	η_1	$\omega_{(S-P)}$	δ_{max}	O_{div}
2	[1 2]	[3 4]	1	4	4	1
	[1 3]	[2 4]	1	4	4	1
	[1 4]	[2 3]	1	3	3	4
	[2 3]	[1 4]	2	4	4	1
	[2 4]	[1 3]	2	3	3	4
	[3 4]	[1 2]	3	2	3	4
3	[1 2 3]	[4]	1	4	4	1
	[1 2 4]	[3]	1	3	3	4
	[1 3 4]	[2]	1	2	2	9
	[2 3 4]	[1]	2	1	2	9

includes the pairs that have $\mathbf{x}_{\eta} = \hat{\mathbf{x}}_{\eta}$ but $\mathbf{x}_{\omega} \neq \hat{\mathbf{x}}_{\omega}$, and the second group comprises the pairs satisfying $\mathbf{x}_{\eta} \neq \hat{\mathbf{x}}_{\eta}$ but $\mathbf{x}_{\omega} = \hat{\mathbf{x}}_{\omega}$. Finally, the last group consists of the pairs for which $\mathbf{x}_{\eta} \neq \hat{\mathbf{x}}_{\eta}$ and $\mathbf{x}_{\omega} \neq \hat{\mathbf{x}}_{\omega}$. We will present the method to calculate the maximum δ for each group, and to find δ_{max} from the groups.

Since the vector $\tilde{\mathbf{d}}$ is a zero vector for the first group, the first summation of κ in (14) is zero, resulting in δ being equal to the minimum of ω . By considering all of the possible pairs, we easily see that $\omega_1 \leq \delta \leq \omega_{(S-P)}$. Therefore, the maximum value is $\delta_1 = \omega_{(S-P)}$ which corresponds to the pair satisfying $x_s = \hat{x}_s$ for all s except $s = \omega_{(S-P)}$. For any pair in the second group, the term with the first singular value survives in κ , according to the inherited property of the constellation precoding matrix, i.e., $|\tilde{d}_1|^2 > 0$. However, the second summation in κ disappears since $\mathbf{x}_{\omega} = \hat{\mathbf{x}}_{\omega}$. Therefore, the maximum value of this group is $\delta_2 = \eta_1$. Now, for the third group, both summations in κ exist. Then, δ is chosen to be the smaller value between the minimum of ω and η_1 . In the same manner as was already given in the analysis of the first group, the maximum of the minimum of ω is found to be $\omega_{(S-P)}$. Therefore, the maximum δ for this group is $\delta_3 = \max\{\eta_1, \omega_{(S-P)}\}$. Finally, δ_{max} is decided as

$$\delta_{max} = \max\{\delta_1, \delta_2, \delta_3\} = \max(\eta_1, \omega_{(S-P)}). \quad (15)$$

Example: We provide the diversity analysis of the 4×4 PPMB system with $S = 4$ and $P = 2$. In this example, we assume that the precoded symbols are transmitted on the subchannel 1 and 3 while the non-precoded symbols are transmitted on the subchannel 2 and 4. Then, this configuration gives $\eta = [1 \ 3]$, and $\omega = [2 \ 4]$. By following the result in (15), δ_{max} is equal to $\max(1, 4) = 4$, leading to the diversity order of 1. The pairwise errors, satisfying $x_1 = \hat{x}_1, x_2 = \hat{x}_2, x_3 = \hat{x}_3$, but $x_4 \neq \hat{x}_4$, inflict loss on the diversity order of this system. Table I summarizes the diversity order analysis for all of the possible combinations of the 4×4 PPMB system. We will provide simulation results that verify this analysis in Section VI, specifically in Fig. 4.

IV. DIVERSITY ANALYSIS : BICMB-CP

A. BICMB with Full Precoding

We assume that the d_H coded bits are interleaved such that they are placed in distinct symbols, where d_H denotes the Hamming distance between the transmitted codeword \mathbf{c} and the decoded codeword $\hat{\mathbf{c}}$. Since the bit metrics in (7) are the same for the same coded bits between the pairwise errors, the original PEP is replaced by

$$\Pr(\mathbf{c} \rightarrow \hat{\mathbf{c}}|\mathbf{H}) = \Pr\left(\sum_{k,d_H} \min_{\mathbf{x} \in \xi_{c_k'}^{l,i}} \|\mathbf{r}_k - \mathbf{\Gamma}\mathbf{\Theta}\mathbf{x}\|^2 \geq \sum_{k,d_H} \min_{\mathbf{x} \in \xi_{\hat{c}_k'}^{l,i}} \|\mathbf{r}_k - \mathbf{\Gamma}\mathbf{\Theta}\mathbf{x}\|^2\right) \quad (16)$$

where the summation is restricted to the symbols corresponding to the different d_H coded bits.

This expression can be upper bounded and then the average PEP can be calculated as [27]

$$\Pr(\mathbf{c} \rightarrow \hat{\mathbf{c}}) \leq E \left[\exp \left(- \frac{\sum_{s=1}^S \lambda_s^2 \sum_{k,d_H} |d_{k,s}|^2}{4N_0} \right) \right] \quad (17)$$

where $d_{k,s}$ is the s^{th} entry of the vector $\mathbf{d}_k = \mathbf{\Theta}(\mathbf{x}_k - \hat{\mathbf{x}}_k)$, $\hat{\mathbf{x}}_k$ is

$$\hat{\mathbf{x}}_k = \arg \min_{\mathbf{x} \in \xi_{c_k'}^{l,i}} \|\mathbf{r}_k - \mathbf{\Gamma}\mathbf{\Theta}\mathbf{x}\|^2, \quad (18)$$

and $\bar{c}_{k'}$ is the complement of $c_{k'}$ in binary codes.

According to Theorem 1, we can evaluate the diversity order of a given system by calculating the weight vector whose s^{th} element is $\sum_{k,d_H} |d_{k,s}|^2$. In particular, if the constellation precoder is designed such that

$$|d_{k,1}|^2 = |\theta_1^T(\mathbf{x}_k - \hat{\mathbf{x}}_k)|^2 > 0, \forall(\mathbf{x}_k, \hat{\mathbf{x}}_k) \quad (19)$$

where θ_1^T is the first row vector of the precoding matrix $\mathbf{\Theta}$, we see that $\sum_{k,d_H} |d_{k,1}|^2 > 0$, resulting in the full diversity order of NM . Therefore, (19) is a sufficient condition for the full diversity order of BICMB-PP.

B. BICMB with Partial Precoding

The average PEP can be calculated as [27]

$$\Pr(\mathbf{c} \rightarrow \hat{\mathbf{c}}) \leq E \left[\frac{1}{2} \exp \left(- \frac{\sigma}{4N_0} \right) \right]. \quad (20)$$

In this expression,

$$\sigma = \sum_{r=1}^P \lambda_{\eta_r}^2 \sum_{k,d_H} |\hat{d}_{k,r}|^2 + d_{min}^2 \sum_{r=1}^{S-P} \lambda_{\omega_r}^2 \alpha_{\omega_r} \quad (21)$$

where $\hat{d}_{k,r}$ is the r^{th} entry of the vector $\hat{\mathbf{d}}_k = \tilde{\mathbf{\Theta}}(\mathbf{x}_{k,\eta} - \hat{\mathbf{x}}_{k,\eta})$, and α_s is the number of times the s^{th} sub-channel is used corresponding to d_H^n bits under consideration.

To determine the diversity order from σ , we need to find the index indicating the first nonzero element in an ordered composite vector which consists of $\sum_{k,d_H} |\hat{d}_{k,r}|^2$ and α_{ω_r} ,

as in Theorem 1. If $d_H^p = 0$, the first summation part of σ vanishes. In this case, the first index is

$$\delta = \min\{s : \alpha_s > 0 \text{ for } s \in \{\omega_1, \dots, \omega_{(S-P)}\}\}. \quad (22)$$

In the other case of $d_H^p > 0$, we see that $\mathbf{x}_{k,\eta}$ and $\hat{\mathbf{x}}_{k,\eta}$ are obviously different for the same reason as in the previous section. If the constellation precoder satisfies the sufficient condition of (19), the term with $\lambda_{\eta_1}^2$ always exists in σ . By considering the second term of σ , we get δ for the case of $d_H^p > 0$

$$\delta = \begin{cases} \min(\eta_1, \delta') & \text{if } \delta' \text{ exists,} \\ \eta_1 & \text{otherwise.} \end{cases} \quad (23)$$

where δ' , if it exists, is obtained in the same way as (22). If, in search of δ' , no s satisfying the right hand side of (22) exists, we state δ' does not exist and set $\delta = \eta_1$, as in (23).

Example: In this example, we employ 4-state 1/2-rate convolutional code with generator polynomials (5,7) in octal representation, in an $N = M = S = 3$ system. Two types of spatial interleavers are used to demonstrate the different results of the diversity order. A generalized transfer function of BICMB with the specific spatial interleaver and convolutional code provides the α -vectors for all of the pairwise errors, whose element indicates the number of times the stream is used for the erroneous bits [8]. In particular, due to the fact that $d_H^p = \sum_{r=1}^P \alpha_{\eta_r}$ and $d_H^n = \sum_{r=1}^{S-P} \alpha_{\omega_r}$ where α_s is the s^{th} element of the α -vector, the generalized transfer function approach in [8] is also useful in the analysis of BICMB-PP. Hence, we rewrite the transfer functions of the systems from [8], where a , b , and c are the symbolic representation of the 1st, 2nd, 3rd streams, respectively. The spatial interleaver used in \mathcal{T}_1 is a simple rotating switch on 3 streams. For \mathcal{T}_2 , the u^{th} coded bit is interleaved into the stream $s_{\text{mod}(u-1,18)+1}$ where $s_1 = \dots = s_6 = 1$, $s_7 = \dots = s_{12} = 2$, $s_{13} = \dots = s_{18} = 3$ and mod is the modulo operation. Each term represents an α -vector, and the powers of a , b , c in this term indicate the elements of the α -vector corresponding to that term.

$$\begin{aligned} \mathcal{T}_1 = & Z^5(a^2b^2c + a^2bc^2 + ab^2c^2) \\ & + Z^6(a^3b^2c + a^2b^3c + a^3bc^2 + \\ & \quad ab^3c^2 + a^2bc^3 + ab^2c^3) \\ & + Z^7(2a^3b^3c + 2a^3b^2c^2 + 2a^2b^3c^2 + \\ & \quad 2a^3bc^3 + 2a^2b^2c^3 + 2ab^3c^3) \\ & + Z^8(a^5b^3 + a^4b^3c + a^3b^4c + 2a^4b^2c^2 + \\ & \quad 3a^3b^3c^2 + 2a^2b^4c^2 + a^4bc^3 + 3a^3b^2c^3 + \\ & \quad 3a^2b^3c^3 + ab^4c^3 + b^5c^3 + a^3bc^4 + \\ & \quad 2a^2b^2c^4 + ab^3c^4 + a^3c^5) + \dots \end{aligned} \quad (24)$$

$$\begin{aligned} \mathcal{T}_2 = & Z^5(a^5 + a^3b^2 + a^2b^3 + \\ & \quad b^5 + a^3c^2 + b^3c^2 + a^2c^3 + b^2c^3 + c^5) \\ & + Z^6(a^4b^2 + 3a^3b^3 + a^2b^4 + a^4c^2 + 3a^2b^2c^2 + \\ & \quad b^4c^2 + 3a^3c^3 + 3b^3c^3 + a^2c^4 + b^2c^4) \\ & + Z^7(2a^4b^3 + 2a^3b^4 + a^3b^3c + 7a^2b^2c^2 + \\ & \quad 7a^2b^3c^2 + 2a^4c^3 + a^3bc^3 + 7a^2b^2c^3 + \\ & \quad ab^3c^3 + 2b^4c^3 + 2a^3c^4 + 2b^3c^4) + \dots \end{aligned} \quad (25)$$

Consider the case $\eta = [1\ 2]$. We see that all of the α -vectors of \mathcal{T}_1 have $d_H^p > 0$. Since $\eta_1 = 1$, δ equals 1 whether δ' exists or not. In fact, δ' does not exist for the term $Z^8 a^5 b^3$. Therefore, the \mathcal{T}_1 BICMB-PP system with $\eta = [1\ 2]$ achieves the full diversity order while BICMB without constellation precoding [8], or PPMB without Bit-Interleaved Coded Modulation (BICM) loses the full diversity order [25], [26]. For \mathcal{T}_2 , the α -vector $[0\ 0\ 5]$ gives $d_H^p = 0$, resulting in $\delta = 3$. Therefore, the \mathcal{T}_2 BICMB-PP system with $\eta = [1\ 2]$ does not achieve the full diversity order.

The same analysis for $\eta = [1\ 3]$ results in the diversity order of 9, and $[2\ 3]$ results in 4 for the transfer function \mathcal{T}_1 . Similarly, both of $[1\ 3]$ and $[2\ 3]$ result in the diversity of 4 for \mathcal{T}_2 . As a consequence, we find that proper selection of the subchannels for precoding, as well as the appropriate pattern of the spatial interleaver, is important to achieve the full diversity order of BICMB-PP. We will present simulation results that verify this analysis in Section VI, in particular, in Fig. 7.

V. REDUCED COMPUTATIONAL COMPLEXITY SPHERE DETECTION

In this section, we will describe the reduced computational complexity sphere detection for constellation precoded multiple beamforming with square QAM modulation. More specifically, we propose the sphere detection technique to reduce the number of multiplications without losing the performance. Since detecting the transmitted non-precoded symbols for UMB-CP in (5) and finding the bit metrics of non-precoded symbols for BICMB-CP in (8) can be carried out independently of the symbols on the other subchannels, we focus on the precoded P symbols.

Solving (4) for MLD is well-known to be NP-hard, given that a full search over the entire lattice space is performed [28]. SD, on the other hand, solves (4) by searching only lattice points that lie inside a sphere of radius ρ centering around the received vector \mathbf{y}_p . A frequently used solution for the QAM-modulated complex signal model is to decompose the P -dimensional complex-valued problem (4) into a $2P$ -dimensional real-valued problem, which is written as

$$\begin{aligned} \bar{\mathbf{y}} &= \begin{bmatrix} \Re\{\mathbf{y}_p\} \\ \Im\{\mathbf{y}_p\} \end{bmatrix} = \bar{\mathbf{F}}\bar{\mathbf{x}} + \bar{\mathbf{n}} \\ &= \begin{bmatrix} \Re\{\mathbf{F}\} & -\Im\{\mathbf{F}\} \\ \Im\{\mathbf{F}\} & \Re\{\mathbf{F}\} \end{bmatrix} \begin{bmatrix} \Re\{\mathbf{x}_\eta\} \\ \Im\{\mathbf{x}_\eta\} \end{bmatrix} + \begin{bmatrix} \Re\{\mathbf{n}_p\} \\ \Im\{\mathbf{n}_p\} \end{bmatrix} \end{aligned} \quad (26)$$

where $\mathbf{F} = \Gamma_p \bar{\mathbf{\Theta}}$ [15], [28]. The QR decomposition of the $2P \times 2P$ real-valued channel matrix turns (4) into the equivalent expression

$$\hat{\mathbf{x}}_\eta = \arg \min_{\mathbf{x} \in \Psi} \|\bar{\mathbf{Q}}^H \bar{\mathbf{y}} - \bar{\mathbf{R}}\mathbf{x}\|^2 \quad (27)$$

where $\bar{\mathbf{Q}}$ and $\bar{\mathbf{R}}$ are the unitary matrix and the upper triangular matrix from the QR decomposition of $\bar{\mathbf{F}}$ [15], [28]. Let Ω denote the set of scalar symbols for one dimension of QAM, e.g., $\Omega = \{-3, -1, 1, 3\}$ for 16-QAM, then Ψ denotes a subset of Ω^{2P} whose elements satisfy $\|\bar{\mathbf{Q}}^H \bar{\mathbf{y}} - \bar{\mathbf{R}}\mathbf{x}\|^2 < \rho^2$. The initial radius ρ should be chosen properly so that it is neither too small nor too large. Too small an initial radius can result in too many unsuccessful searches by restarting the search

and thus increasing the complexity, while too large an initial radius can result in too many lattice points to be searched.

The SD algorithm can be viewed as a pruning algorithm on a tree of depth $2P$, whose branches correspond to elements drawn from the set Ω [23], [28]. Conventional SD implements a Depth-First Search (DFS) strategy in the tree which achieves MLD performance. The complexity of SD is measured in terms of the number of operations required per visited node multiplied by the number of visited nodes throughout the search algorithm [28]. The complexity can be reduced by either reducing the number of nodes to be visited or the number of operations to be carried out at each node or both. In order to reduce the number of visited nodes, one can either make a judicious choice of the initial radius to start the algorithm, or execute a proper sphere radius update strategy. The former strategy has been studied in [16] and [17], and the latter one has been discussed in [18] and [19]. In this paper, we propose methods to reduce the average number of real multiplications, which are the most expensive operations in terms of machine cycles required at each node for conventional SD. A proper choice of the initial radius for BICMB-CP will also be provided.

We start by writing the node weight as [23]

$$w(\bar{\mathbf{x}}^{(u)}) = w(\bar{\mathbf{x}}^{(u+1)}) + w_{pw}(\bar{\mathbf{x}}^{(u)}) \quad (28)$$

with $u = 2P, 2P-1, \dots, 1$, $w(\bar{\mathbf{x}}^{(2P+1)}) = 0$, and $w_{pw}(\bar{\mathbf{x}}^{(2P+1)}) = 0$, where $\bar{\mathbf{x}}^{(u)}$ denotes the partial vector symbol at layer u . The partial weight $w(\bar{\mathbf{x}}^{(u)})$ is written as

$$w_{pw}(\bar{\mathbf{x}}^{(u)}) = |\tilde{y}_u - \sum_{v=u}^{2P} \bar{R}_{u,v} \bar{x}_v|^2 \quad (29)$$

where \tilde{y}_u is the u^{th} element of $\bar{\mathbf{Q}}^H \bar{\mathbf{y}}$, $\bar{R}_{u,v}$ is the $(u, v)^{th}$ element of $\bar{\mathbf{R}}$, and \bar{x}_v is the v^{th} element of $\bar{\mathbf{x}}$.

A. Precalculation of Multiplications

Note that for one channel realization, both $\bar{\mathbf{R}}$ and Ω are independent of time. In other words, to decode different received symbols for one channel realization, the only term in (29) which depends on time is \tilde{y}_u . Consequently, a table \mathbb{T} can be constructed to store all terms of $\bar{R}_{u,v} \bar{x}$, where $\bar{R}_{u,v} \neq 0$ and $\bar{x} \in \Omega$, before starting the tree search procedure. Equations (28) and (29) imply that only one real multiplication is needed by using \mathbb{T} instead of $2P-u+2$ for each node to calculate the node weight. As a result, the number of real multiplications can be significantly reduced.

Taking the square QAM structure into consideration, Ω can be divided into two smaller sets Ω_1 with negative elements and Ω_2 with positive elements. Take 16-QAM for example, $\Omega = \{-3, -1, 1, 3\}$, then $\Omega_1 = \{-3, -1\}$ and $\Omega_2 = \{1, 3\}$. Any negative element in Ω_1 has a positive element with the same absolute value in Ω_2 . Consequently, in order to build \mathbb{T} , only terms of $\bar{R}_{u,v} \bar{x}$, where $\bar{R}_{u,v} \neq 0$ and $\bar{x} \in \Omega_1$, need to be calculated and stored. Hence, the size of \mathbb{T} is

$$|\mathbb{T}| = \frac{N_R |\Omega|}{2} \quad (30)$$

where N_R denotes the number of nonzero elements in matrix $\bar{\mathbf{R}}$, and $|\Omega|$ denotes the size of Ω .

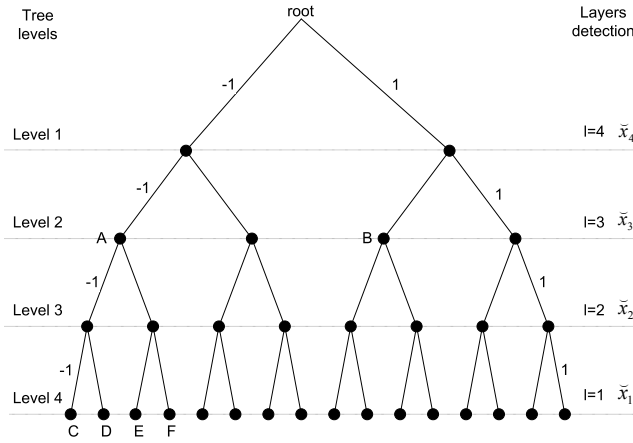


Fig. 2. Tree structure for a 2×2 FPMB system employing 4-QAM.

In order to build \mathbb{T} , both the number of terms that need to be stored and the number of real multiplications required are $|\mathbb{T}|$. Since the channel is assumed to be flat fading, only one \mathbb{T} needs to be built in one burst. If the burst length is very long, the computational complexity of building \mathbb{T} can be neglected.

B. Modified DFS Algorithm

The representation proposed in [23] replaces the conventional representation of (26) with

$$\tilde{\mathbf{y}} = \mathbf{G}\tilde{\mathbf{x}} + \tilde{\mathbf{n}} \quad (31)$$

where $\tilde{\mathbf{y}}$, $\tilde{\mathbf{x}}$, and $\tilde{\mathbf{n}}$ consist of the real and imaginary parts of the members of $\bar{\mathbf{y}}$, $\bar{\mathbf{x}}$, and $\bar{\mathbf{n}}$, respectively, and \mathbf{G} has the corresponding real and imaginary parts of the members of \mathbf{F} . The structure of the lattice representation becomes advantageous after applying the QR decomposition to \mathbf{G} , i.e., $\mathbf{G} = \mathbf{Q}\mathbf{R}$. Due to a special form of orthogonality between each pair of columns, all elements $R_{u,u+1}$ for $u = 1, 3, \dots, 2P - 1$, in the upper triangular matrix \mathbf{R} become zero [23]. The locations of these zeros introduce orthogonality between the real and imaginary parts of every detected symbol, which can be taken advantage of to reduce the computational complexity of SD. We provide the following example to explain this.

Consider a 2×2 $S = 2$ FPMB system employing 4-QAM. Then, SD constructs a tree with $2P = 4$ levels, where the branches coming out of each node represent the real values in the set $\Omega = \{-1, 1\}$. This tree is shown in Fig. 2.

Calculating partial node weights for the first level and the second level are independent, same as the third level and the fourth level, because of the additional zeros in the \mathbf{R} matrix. For instance, the partial weights of node *A* and *B* in Fig. 2 depend only on \tilde{x}_3 , and the partial weights of node *C*, *D*, *E*, and *F* depend on \tilde{x}_4 , \tilde{x}_3 , and \tilde{x}_1 except \tilde{x}_2 . In other words, the partial weights of node *A* and *B* are equal, and need to be calculated once. Similarly, partial weights of node *C* and *D* can be used without an additional computation for the partial weights of node *E* and *F*, respectively.

Because of this feature, the DFS strategy is modified in the following way: for the u^{th} layer, where u is an odd number, partial weights of the nodes at the layer u (called

children nodes) belonging to a node at the layer $u + 1$ (called *a parent node*) are stored, and are used as partial weights of the nodes belonging to the same node at the layer $u + 2$ (called *a grandparent node*), but to the different parent nodes. In other words, the weights of children nodes belonging to one of the parent nodes are recycled by the children's *cousins*.

By implementing the modified DFS algorithm, further complexity reduction is achieved beyond the reduction due to the precalculation table \mathbb{T} [27].

C. Initial Radius for BICMB-CP

The proposed SD algorithm for UMB-CP described in the previous sections can also be applied to BICMB-CP. However, a straightforward implementation of this algorithm can result in unsuccessful searches for bit metrics which in turn results in unnecessary complexity. To solve this problem, we used an initial radius determined by the ZF-DFE algorithm. With the initial radius acquired by the ZF-DFE algorithm, the SD guarantees no unsuccessful search for the bit metrics. A description of our technique can be found in [27].

D. General Signal Constellations

The square QAM constellation enables the separation of real and imaginary parts of the received signals and results in the simple structure discussed in the previous subsection. This structure results in a substantial reduction in computational complexity as will be illustrated in simulation results. The basic technique is applicable to general constellations such as non-square QAM and MPSK. In the case of non-square QAM, the real and the imaginary parts need to be treated differently depending on their signal set and mapping. For MPSK, the real and imaginary parts cannot be treated separately, and the real-valued SD is not applicable. The computational complexity reduction can be applied to a complex-valued SD. For both non-square QAM and MPSK, the resulting computational complexity will be less than conventional SD, but more than for square QAM.

VI. SIMULATION RESULTS

A. UMB-CP

To illustrate the analysis of the diversity order in Section III, we now present simulation results over a number of different system configurations. Fig. 3 shows BER performance for SB and FPMB. The curves with the legend FPMB are generated by the precoding matrices that outperform the others in [25], [26]. All of the FPMB systems employ 4-QAM modulation, and the system data rate for SB and FPMB is set to 4, 8 bits/channel use for a 2×2 and a 4×4 system, respectively. All of the FPMB systems are shown to achieve the full diversity order since each slope is parallel to the corresponding SB system, known to achieve the full diversity order of NM . We note that a larger number of singular values leads to a bigger array gain [25].

Simulation results to support the diversity analysis of 4×4 $S = 4$ PPMB in Table I are provided in Fig. 4. We find that the simulation results are the same as the diversity orders in Table I.

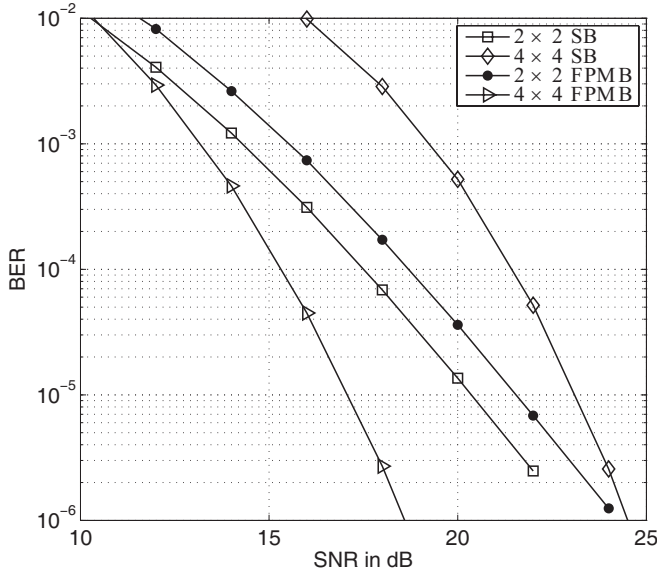


Fig. 3. BER vs. SNR comparison for 2×2 , 4×4 SB and FPMB.

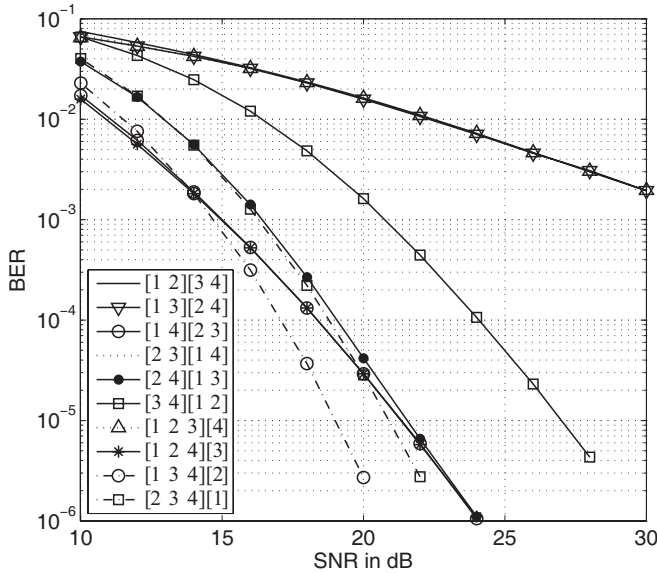


Fig. 4. BER vs. SNR for 4×4 $S = 4$, 4-QAM PPMB.

To verify the reduced computational complexity with sphere detection in Section V, we simulated 2×2 $S = 2$ and 4×4 $S = 4$ FPMB systems using 4-QAM and 64-QAM with receivers employing the exhaustive search (EXH), the conventional SD (CSD), and the proposed SD (PSD). In these simulations, the initial radius is chosen to be $\rho^2 = 2N_0P$, inside which at least one lattice point lies with a high probability [18]. The average number of real multiplications for decoding one transmitted vector symbol is calculated at different SNR. Since the reductions in complexity are substantial, we will express them as orders of magnitude (in approximate terms) in the sequel. We will describe both sets of results, but due to space limitations, provide plots only for 4×4 . Fig. 5 shows the simulation results of 4×4 $S = 4$ FPMB system. For 4-QAM, the number of multiplications of CSD is reduced by 1.4 and 2.1 orders of magnitude at low and high SNR, respectively. PSD reduces the complexity by 2.1 orders of magnitude at

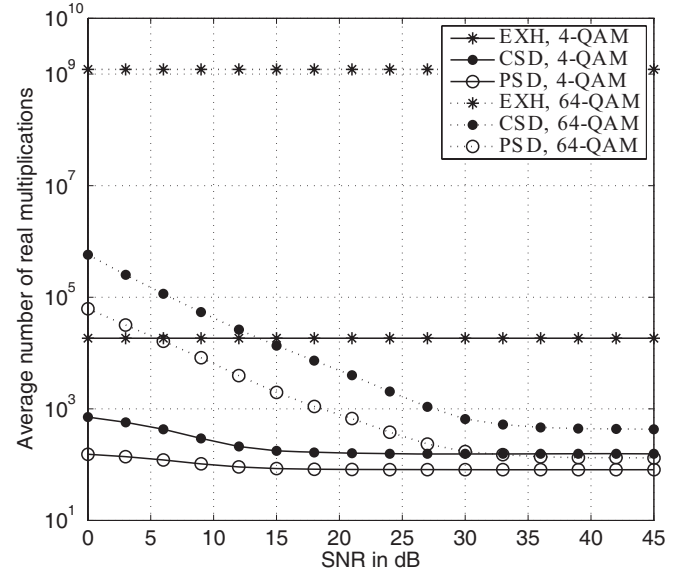


Fig. 5. Average number of real multiplications vs. SNR for the 4×4 FPMB systems with 4-QAM and 64-QAM.

low SNR, and 2.4 at high SNR. The reduction becomes larger as the constellation size increases in the 4×4 $S = 4$ FPMB system. For 64-QAM, the number of multiplications of CSD decreases by 3.3 and 6.4 orders of magnitude at low and high SNR, respectively. PSD gives a larger reduction by 4.3 orders of magnitude at low SNR, and 7.0 at high SNR. In the case of the 2×2 $S = 2$ FPMB system, for 4-QAM, a comparison with EXH shows that CSD reduces the number of multiplications by approximately 0.6 and 0.8 orders of magnitude at low and high SNR, respectively, and PSD reduces by approximately 1.0 and 1.1 order of magnitude at low and high SNR, respectively. For 64-QAM, the reduction in complexity increases: the number of multiplications of CSD decreases by approximately 1.4 orders of magnitude at low SNR, and 2.8 at high SNR, while that of PSD decreases by 2.4 and 3.2 orders of magnitude at low and high SNR, respectively. Simulation results clearly show that CSD reduces the complexity substantially compared with EXH, and the complexity can be further reduced effectively by our PSD. The complexity reduction becomes larger as the constellation precoder dimension or the constellation size becomes larger.

B. BICMB-CP

To verify the diversity analysis in Section IV, Fig. 6 depicts the simulation results for 2×2 , 3×3 , and 4×4 BICMB and BICMB-FP with 64-state convolutional code punctured from 1/2-rate mother code with generator polynomials (133, 171) in octal representation. In [8], we showed the maximum achievable diversity order of BICMB with an R_c -rate convolutional code is $(N - \lceil S \cdot R_c \rceil + 1)(M - \lceil S \cdot R_c \rceil + 1)$. In this example, the maximum achievable diversity order of the three BICMB systems is 1. However, Fig. 6 shows that BICMB-FP achieves the full diversity order for any code rate.

Fig. 7 depicts the simulation results of BICMB-PP given in the example of Section III-B. The diversity orders of the BICMB systems, \mathcal{T}_1 and \mathcal{T}_2 are 4 and 1, respectively [8].

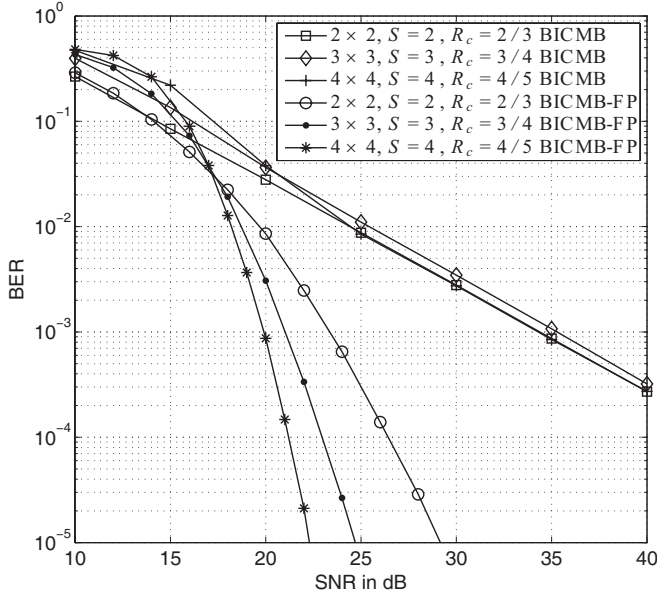


Fig. 6. BER comparison between BICMB and BICMB-FP with 16-QAM, and 64-state punctured convolutional code.

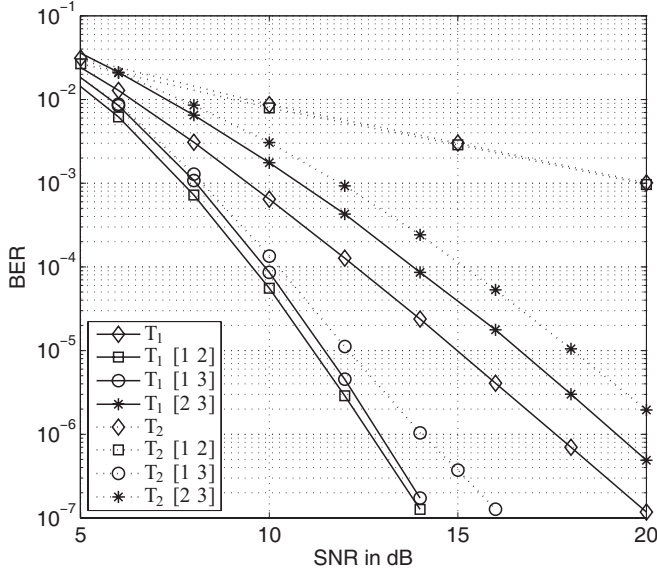


Fig. 7. BER vs. SNR for BICMB-PP with 3×3 $S = 3$, 4-QAM, and 4-state $1/2$ -rate convolutional code.

Comparing the slopes of BICMB-PP with BICMB, we see that the simulation results match the analysis in Section III-B.

To verify the proposed sphere decoding technique in this case for BICMB-FP, we simulated 2×2 $S = 2$, 64-state $R_c = 2/3$ BICMB-FP systems, and 4×4 $S = 4$, 64-state $R_c = 4/5$ BICMB-FP systems using 4-QAM and 64-QAM modulation with Gray mapping. The average number of real multiplications for acquiring one bit metric is calculated with receivers employing EXH, CSD, and PSD. Initial radii for both of CSD and PSD are determined by the ZF-DFE algorithm. As in the previous subsection, we will describe both sets of results, but provide plots only for 4×4 . Fig. 8 shows the number of multiplications of CSD for 4-QAM decreases by 1.3 and 1.5 orders of magnitude at low and

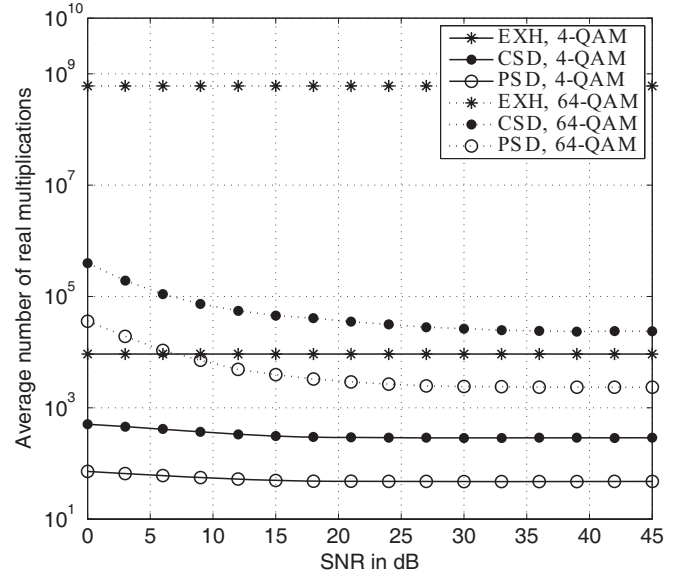


Fig. 8. Average number of real multiplications vs. SNR for the 4×4 BICMB-FP systems with 4-QAM and 64-QAM.

high SNR, respectively. PSD gives bigger reductions by 2.1 orders of magnitude at low SNR, and 2.3 at high SNR. For the 64-QAM case, reductions between EXH and CSD by 3.2 and 4.4 orders of magnitude are observed at low and high SNR, respectively, while larger reductions by 4.2 and 5.4 are achieved by PSD. In the case of the 2×2 BICMB-FP system, we observe that the number of multiplications of CSD for 4-QAM is reduced by 0.4 and 0.5 orders of magnitude at low and high SNR, respectively. PSD yields bigger reductions by 1.0 and 1.1 orders of magnitude at low and high SNR, respectively. In the case of 64-QAM, reductions between CSD and EXH are 1.5 and 2.1 orders of magnitude at low and high SNR, respectively, while larger reductions of 2.4 and 2.9 are achieved by PSD. Similarly to the uncoded case, the complexity reduction becomes larger as the constellation precoder dimension or the constellation size becomes larger. One important property of our decoding technique needs to be emphasized: the substantial complexity reduction achieved causes no performance degradation.

VII. FURTHER REDUCTION IN DECODER COMPLEXITY

Our goal in this paper has been to show that the BICMB structure with precoding can achieve the maximum spatial multiplexing together with the maximum diversity order offered by the MIMO channel. We have shown that this is possible via the MLD. To simplify the complexity of MLD we have shown that by using SD, one can reduce the complexity of MLD by several orders of magnitude. We note that the expected complexity of SD has been studied in detail in the literature. In [15], it has been shown that although SD can be efficient for some SNR and problems of moderate size, its complexity actually grows as an exponential function of the problem size. Although we have shown that a substantial complexity reduction is achievable via SD, in the sequel we will discuss recent developments in the literature that can help reduce the complexity even further.

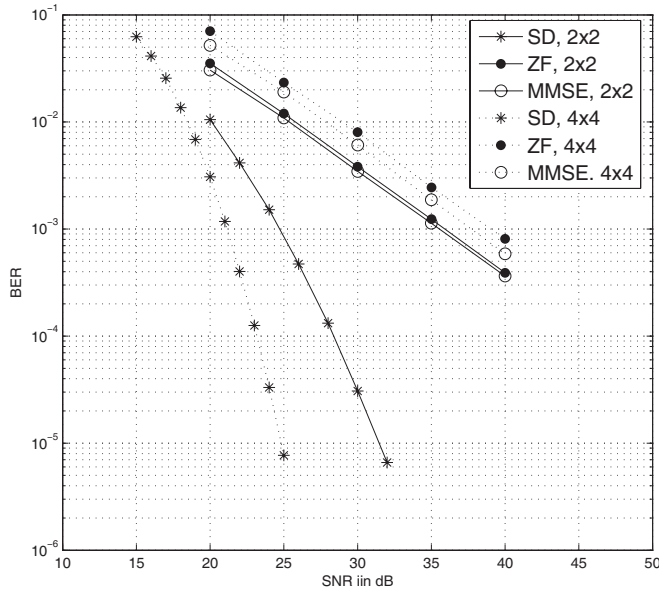


Fig. 9. BER comparison of FPMB with SD, MMSE, and ZF decoders and 16-QAM.

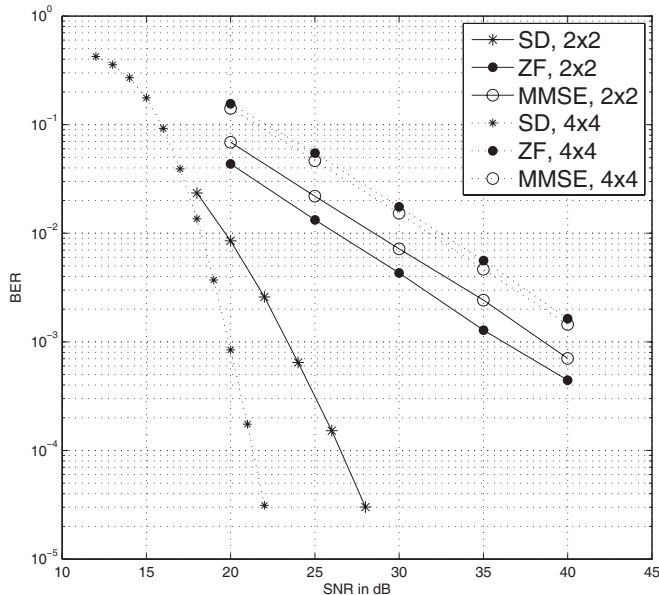


Fig. 10. BER comparison of BICMB-CP with SD, MMSE, and ZF decoders and 16-QAM.

Simple decoders such as Minimum Mean Squared Error (MMSE) or Zero-Forcing (ZF) will not be able to capture the diversity order or the performance of SD. As examples, Fig. 9 depicts the performance of FPMB and Fig. 10 depicts the performance of BICMB-CP versus MMSE and ZF decoding.

Within the last few years, a large number of approaches that attack the complexity of decoding in MIMO systems have been published. These can be classified roughly into three categories. The first category consists of techniques that reduce the complexity of SD. For example, the approach in [29] is based on searching a partitioned symbol vector space rather than that spanned by the whole symbol vector space. This is equivalent to carrying out the search in a reduced dimension space. There is an inevitable performance loss due to this

approach but this loss is compensated by a more sophisticated tree search and the recomputation of a set of symbols ignored in the reduced dimension search. Simulations for large $N \times N$ MIMO systems at high SNR show nearly constant complexity over a wide range of BER values with performance limited to 1 dB of MLD. Complexity reductions with respect to SD are modest when compared with the order-of-magnitude reductions with SD against MLD. In [30], a fixed-complexity SD is introduced. It is proven that this decoder achieves the same diversity order, and at high SNR, same performance as MLD. However, the technique is specified for uncoded MIMO systems. A number of other techniques for reducing the complexity of SD exist. Examples are Schnorr-Euchner enumeration, e.g., [31], [32], radius adaptive or increasing radii SD, e.g., [33], [34].

A second group of approaches employ tree search techniques different than SD for low-complexity MIMO decoding, and can have practical interest. In [35], an augmented channel matrix approach that reduces the lattice search is introduced. It has been proven that this approach provides the maximum receive diversity offered by the MIMO channel. Simulation results show that for large $N \times N$, the system outperforms another well-known lattice reduction technique, Lenstra-Lenstra-Lovasz (LLL) algorithm followed by successive interference cancellation with a slight increase in complexity. In [36], an approach from error correction coding, the Chase algorithm, has been adopted for MIMO detection. Some complexity reduction with a slight degradation in performance is possible by using this technique. However, it has been shown in [29] that further complexity reduction than [36] is possible by the technique in [29]. We count the approach of adopting sequential decoding algorithms to MIMO decoding in this second category. Sequential decoding algorithms, such as the Fano and the stack algorithms were used to decode convolutional codes prior to the discovery of the Viterbi algorithm. For example, [37] and [38] adapt the stack algorithm to MIMO decoding, [39] parallelizes the stack algorithm for MIMO, and [40] proposes a multistack algorithm for soft MIMO decoding. Another group of techniques employ randomized tree search. These are known as Monte Carlo Tree Search [41] or Markov Chain Monte Carlo [42], [43] techniques.

A third group of algorithms uses a variety of techniques to achieve MIMO decoding with a reduction in complexity, trading off performance in many cases. For example, [44] uses an approach of first MMSE decoding and then choosing among a small number of candidates via a reliability metric. This approach achieves the same performance as MLD with about 28% (not orders of magnitude) reduction in complexity. Reference [45] employs simplifications in bit constellation mapping in QAM, with complexity reductions reported up to one degree of magnitude for 64-QAM. Some researchers employed a variety of mathematical programming techniques for MIMO decoding. For example [46] employed linear and mixed integer linear programming with the L_1 norm, and [47] used semi-definite programming.

Examples of different approaches to MLD can be increased. However, after a close examination of these approaches, it can be stated that the orders of magnitude reduction by SD over MLD as well as our particular approach over SD remain

substantial. While an alternative to the approach described in the previous section in terms of performance and complexity is not immediately clear, some of the proposed techniques can be employed to provide additional complexity reductions, albeit unlikely to be by orders of magnitude. Examples of such techniques are [29], possibly a version of [30] for coded systems, and [45].

VIII. CONCLUSION

In this paper, we proposed constellation precoded multiple beamforming which achieves the full diversity order in both of the uncoded and coded MIMO multiple beamforming systems when the channel information is perfectly available at the transmitter as well as the receiver, at different levels of spatial multiplexing, including the maximum ($\min(N, M)$) provided by the $N \times M$ channel. Diversity analysis was given in both of the multiple beamforming schemes through the calculation of pairwise error probability. We provided examples of calculating the diversity orders of various multiple beamforming systems and simulation results supporting the analysis. A sphere detection algorithm which improves the complexity was proposed so that constellation precoded multiple beamforming can be considered as a practical implementation for MIMO systems requiring high throughput with the full diversity order. The proposed SD algorithm in this paper can be applied to any MIMO system.

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Hong Ju Park received the B.S. and M.S. degrees in electronics engineering from Sogang University, Seoul, Korea, in 1994 and 1996, respectively. He completed the Ph.D. degree in electrical and computer engineering at the University of California, Irvine, in 2010. From 1996 to 2005, he was with the Mobile Device Division, Samsung Electronics, Suwon, Korea, where he worked on the firmware design of GSM/GPRS/EDGE mobile phones with emphasis on the air interface protocol. From 2005 to 2010, he was with the Center for Pervasive

Communications and Computing, University of California, Irvine, where he was a Graduate Student Researcher. After returning to the previous division at Samsung Electronics in 2010, he has been working as a Principal Engineer, concentrating on system design of current and next generation wireless products. His research interests include space-time coding, beamforming and MIMO-OFDM.



Boyu Li received the B.S. degree from Beijing Institute of Technology, Beijing, China, in 2007, and the M.S. degree from University of California, Irvine, Irvine, CA, in 2009, respectively, both in electrical engineering. He is currently working toward the Ph.D. degree with the Department of Electrical Engineering and Computer Science, University of California. From 2009, he has been a Graduate Student Researcher with the Center for Pervasive Communications and Computing, University of California, Irvine. His current research interests include full spatial multiplexing and full diversity MIMO systems with low complexity implementation, as well as single- and multi-microphone dereverberation.



Ender Ayanoglu (S'82-M'85-SM'90-F'98) received the B.S. degree from the Middle East Technical University, Ankara, Turkey, in 1980, and the M.S. and Ph.D. degrees from Stanford University, Stanford, CA, in 1982 and 1986, respectively, all in electrical engineering. He was with the Communications Systems Research Laboratory, AT&T Bell Laboratories, Holmdel, NJ (Bell Labs, Lucent Technologies after 1996) until 1999 and was with Cisco Systems until 2002. Since 2002, he has been a Professor in the Department of Electrical Engineering and Computer Science, University of California, Irvine, where he served as the Director of the Center for Pervasive Communications and Computing and held the Conexant-Broadcom Endowed Chair during 2002–2010. Dr. Ayanoglu is the recipient of the IEEE Communications Society Stephen O. Rice Prize Paper Award in 1995 and the IEEE Communications Society Best Tutorial Paper Award in 1997. Since 1993, he has been an Editor of the IEEE TRANSACTIONS ON COMMUNICATIONS and served as its Editor-in-Chief from 2004 to 2008. From 1990 to 2002, he served on the Executive Committee of the IEEE Communications Society Communication Theory Committee, and from 1999 to 2001, was its Chair.

Engineering and Computer Science, University of California, Irvine, where he served as the Director of the Center for Pervasive Communications and Computing and held the Conexant-Broadcom Endowed Chair during 2002–2010. Dr. Ayanoglu is the recipient of the IEEE Communications Society Stephen O. Rice Prize Paper Award in 1995 and the IEEE Communications Society Best Tutorial Paper Award in 1997. Since 1993, he has been an Editor of the IEEE TRANSACTIONS ON COMMUNICATIONS and served as its Editor-in-Chief from 2004 to 2008. From 1990 to 2002, he served on the Executive Committee of the IEEE Communications Society Communication Theory Committee, and from 1999 to 2001, was its Chair.