# Real-Valued Maximum Likelihood Decoder for Quasi-Orthogonal Space-Time Block Codes

Luay Azzam, *Student Member, IEEE,* and Ender Ayanoglu, *Fellow, IEEE*

*Abstract*—In this letter, we propose a low complexity Maximum Likelihood (ML) decoding algorithm for quasi-orthogonal space-time block codes (QOSTBCs) based on the real-valued lattice representation and QR decomposition. We show that for a system with rate $r = n_s/T$, where $n_s$ is the number of transmitted symbols per $T$ time slots; the proposed algorithm decomposes the original complex-valued system into a parallel system with $n_s$ $2 \times 2$ real-valued components, thus allowing for a simple joint decoding of two real symbols. For a square QAM constellation with $L$ points ($L$-QAM), this algorithm achieves full diversity by properly incorporating two-dimensional rotation using the optimal rotation angle and the same rotating matrix for any number of transmit antennas ($N \geq 4$). We show that the complexity gain becomes greater when $N$ or $L$ becomes larger. The complexity of the proposed algorithm is shown to be linear with the number of transmitted symbols $n_s$.

*Index Terms*—Quasi-orthogonal space-time block codes, pairwise symbol decoding, multiple-input multiple-output (MIMO), maximum likelihood (ML) detection, diversity, wireless communications.

## I. INTRODUCTION

**T**HIS letter is on a simplified decoding technique for quasi-orthogonal space-time block codes (QOSTBCs). For a background on decoding QOSTBCs and recent approaches in the literature, we refer the reader to [1]. Our new decoding algorithm is based on QR decomposition of the real-valued lattice representation for the class of QOSTBCs discussed in [2]. Using this representation, we show that ML detection can be performed jointly considering only two real symbols. We further show that the equivalent channel as seen by every two real symbols is $2 \times 2$, upper triangular, and real-valued; a property which substantially reduces the decoding complexity. Full diversity is achieved by properly incorporating two-dimensional rotation. This rotation is performed using the same rotating matrix for any $N$. This becomes possible by an appropriate grouping of the complex transmitted symbols.

## II. QOSTBCs AND SYSTEM MODEL

Consider a MIMO system with $N$ transmit and $M$ receive antennas, and an interval of $T$ symbols during which the channel is constant. The received signal is given by

$$\mathbf{Y} = \sqrt{\frac{\rho}{N}} \mathbf{C H} + \mathbf{V} \tag{1}$$

where $\mathbf{Y} = [y_t^j]_{T \times M}$ is the received signal matrix of size $T \times M$ and whose entry $y_t^j$ is the signal received at antenna $j$ at time $t$, $t = 1, 2, \ldots, T$, $j = 1, 2, \ldots, M$; $\mathbf{V} = [v_t^j]_{T \times M}$ is the noise matrix; and $\mathbf{C} = [c_t^i]_{T \times N}$ is the transmitted signal matrix whose entry $c_t^i$ is the signal transmitted at antenna $i$ at time $t$, $t = 1, 2, \ldots, T$, $i = 1, 2, \ldots, N$. The matrix $\mathbf{H} = [h_{i,j}]_{N \times M}$ is the channel coefficient matrix of size $N \times M$ whose entry $h_{i,j}$ is the channel coefficient from transmit antenna $i$ to receive antenna $j$. The entries of the matrices $\mathbf{H}$ and $\mathbf{V}$ are independent, zero-mean, and circularly symmetric complex Gaussian random variables of unit variance; and the parameter $\rho$ is the signal-to-noise-ratio (SNR) per receiving antenna.

In this letter, we consider the class of QOSTBCs for which decoding pairs of symbols independently is possible (see Chapter 5 of [3]). The analysis in the sequel applies to any QOSTBC that belongs to this class [2], [4]. Due to space limitation, we only consider $N = 4$ and emphasize the fact that the proposed algorithm and the following derivation is applicable to arbitrary $N$. For $N = 4$, choosing any of the full-rate codes presented in [4], [5], or [6] leads to the same analysis. We consider the one presented in [5] with $n_s = 4$.

Assuming that the channel $\mathbf{H}$ is known at the receiver and setting $M = 1$ for simplicity, the ML estimate is obtained at the decoder by performing $\min_{\mathbf{C}} ||\mathbf{Y} - \sqrt{\frac{\rho}{N}} \mathbf{C H}||_F^2$, where $||.||_F$ is the Frobenius norm. For all QOSTBCs, the measure $||\mathbf{Y} - \sqrt{\frac{\rho}{N}} \mathbf{C H}||_F^2$ can be decoupled into two parts, where each part solves $N/2$ symbols concurrently [7]. However, the complexity can still be reduced substantially as will become clearer in the sequel.

## III. PROPOSED ALGORITHM

We start by decomposing the $T$-dimensional complex problem defined by (1) to a $2T$-dimensional real-valued problem. Using either of the real-valued representations proposed in [5] or [8] gives the same results in terms of the proposed algorithm. Recalling that $M = 1$, and rewriting (1) in matrix form, we have

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_T \end{bmatrix} = \mathbf{C}_N \begin{bmatrix} h_{1,1} \\ h_{2,1} \\ \vdots \\ h_{N,1} \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_T \end{bmatrix}. \tag{2}$$

Note that if $M \neq 1$, then the matrices $\mathbf{Y}$, $\mathbf{H}$, and $\mathbf{V}$ are rearranged to a one column vector by stacking their columns one after the other [9], and $\mathbf{C}_N$ is replaced by $\check{\mathbf{C}}_N$ where $\check{\mathbf{C}}_N \triangleq \mathbf{I}_M \otimes \mathbf{C}_N$, with $\mathbf{I}_M$ is the identity matrix of size $M$ and $\otimes$ denoting the Kronecker matrix multiplication [9].

Now, we specify the complex transmitted symbols of $\mathbf{C}_N$ by their real and imaginary parts as $s_m = x_{2m-1} + j x_{2m}$ for $m = 1, 2, 3, 4$. Applying the real-valued lattice representation

defined in [8] to (2), we obtain

$$\check{\mathbf{y}} = \check{\mathbf{H}}\check{\mathbf{x}} + \check{\mathbf{v}} \qquad (3)$$

or equivalently

$$\begin{bmatrix} \Re(y_1) \\ \Im(y_1) \\ \vdots \\ \Re(y_T) \\ \Im(y_T) \end{bmatrix} = \check{\mathbf{H}} \begin{bmatrix} \Re(s_1) \\ \Im(s_1) \\ \vdots \\ \Re(s_m) \\ \Im(s_m) \end{bmatrix} + \begin{bmatrix} \Re(v_1) \\ \Im(v_1) \\ \vdots \\ \Re(v_T) \\ \Im(v_T) \end{bmatrix}. \qquad (4)$$

The real-valued fading coefficients of $\check{\mathbf{H}}$ are defined using the complex fading coefficients $h_{i,j}$ from transmit antenna $i$ to receive antenna $j$ as $h_{2n-1} = \Re(h_{n,1})$, and $h_{2n} = \Im(h_{n,1})$ for $n = 1, 2, \ldots, N$ (recall that we restricted ourselves to $M = 1$, and therefore we only consider $j = 1$). Let's define the number of transmitted symbols as $n_s$ where $n_s = 4$ for $N = T = 4$. Then, the equivalent real-valued channel $\check{\mathbf{H}}$ is a $2N \times 2n_s$ matrix and is defined as

$$\check{\mathbf{H}} = \begin{bmatrix} h_1 & -h_2 & h_7 & -h_8 & h_3 & -h_4 & h_5 & -h_6 \\ h_2 & h_1 & h_8 & h_7 & h_4 & h_3 & h_6 & h_5 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ h_8 & h_7 & h_2 & h_1 & -h_6 & -h_5 & -h_4 & -h_3 \end{bmatrix}. \qquad (5)$$

Setting $\check{\mathbf{H}} = \begin{bmatrix} \check{\mathbf{h}}_1 & \check{\mathbf{h}}_2 & \cdots & \check{\mathbf{h}}_8 \end{bmatrix}$, where $\check{\mathbf{h}}_k$ is the $k$-th column of $\check{\mathbf{H}}$, we have

$$\langle \check{\mathbf{h}}_1, \check{\mathbf{h}}_i \rangle = 0, i \neq 3, \qquad \langle \check{\mathbf{h}}_2, \check{\mathbf{h}}_i \rangle = 0, i \neq 4$$
$$\langle \check{\mathbf{h}}_5, \check{\mathbf{h}}_i \rangle = 0, i \neq 7, \qquad \langle \check{\mathbf{h}}_6, \check{\mathbf{h}}_i \rangle = 0, i \neq 8. \qquad (6)$$

where $\langle \check{\mathbf{h}}_i, \check{\mathbf{h}}_j \rangle$ is the inner product of columns $\check{\mathbf{h}}_i$ and $\check{\mathbf{h}}_j$. Interchanging the columns of $\check{\mathbf{H}}$ such that every two columns that are not orthogonal to each other become adjacent makes the subsequent analysis appear in a compact form. Thus, we rewrite (3) as

$$\tilde{\mathbf{y}} = \tilde{\mathbf{H}}\mathbf{x} + \tilde{\mathbf{v}} \qquad (7)$$

where $\tilde{\mathbf{H}} = \begin{bmatrix} \check{\mathbf{h}}_1 & \check{\mathbf{h}}_3 & \check{\mathbf{h}}_2 & \check{\mathbf{h}}_4 & \check{\mathbf{h}}_5 & \check{\mathbf{h}}_7 & \check{\mathbf{h}}_6 & \check{\mathbf{h}}_8 \end{bmatrix}$. Consequently, the vectors $\tilde{\mathbf{y}}$ and $\tilde{\mathbf{v}}$ in (7) are the same as $\check{\mathbf{y}}$ and $\check{\mathbf{v}}$ in (3), and the vector $\mathbf{x}$ is the same as the vector $\check{\mathbf{x}}$ but with different order of elements given by $1, 3, 2, 4, 5, 7, 6, 8$. For example, $x_2 = \check{x}_3$. We rewrite (6) using the new order of $\tilde{\mathbf{H}}$ as

$$\langle \tilde{\mathbf{h}}_1, \tilde{\mathbf{h}}_i \rangle = 0, i \neq 2, \qquad \langle \tilde{\mathbf{h}}_5, \tilde{\mathbf{h}}_i \rangle = 0, i \neq 6$$
$$\langle \tilde{\mathbf{h}}_3, \tilde{\mathbf{h}}_i \rangle = 0, i \neq 4, \qquad \langle \tilde{\mathbf{h}}_7, \tilde{\mathbf{h}}_i \rangle = 0, i \neq 8. \qquad (8)$$

Applying QR decomposition to (7), we have

$$\tilde{\mathbf{y}} = \mathbf{QRx} + \tilde{\mathbf{v}}$$
$$\mathbf{Q}^H\tilde{\mathbf{y}} = \mathbf{Rx} + \mathbf{Q}^H\tilde{\mathbf{v}}$$
$$\bar{\mathbf{y}} = \mathbf{Rx} + \bar{\mathbf{v}} \qquad (9)$$

where $\bar{\mathbf{v}}$ and $\tilde{\mathbf{v}}$ have the same statistical properties since $\mathbf{Q}$ is unitary and so is $\mathbf{Q}^H$. Writing this in a matrix form produces a block diagonal matrix $\mathbf{R}$ of the form $\mathbf{R} = \text{diag}(\mathbf{R}_{1,2}, \mathbf{R}_{3,4}, \mathbf{R}_{5,6}, \mathbf{R}_{7,8})$, where

$$\mathbf{R}_{i,i+1} = \begin{bmatrix} r_{i,i} & r_{i,i+1} \\ 0 & r_{i+1,i+1} \end{bmatrix} \qquad i = 1, 3, 5, 7. \qquad (10)$$

Note that the elements of the upper triangular matrix $\mathbf{R}$ are the inner products of columns of $\tilde{\mathbf{H}}$, and due to the properties given in (8), it is straightforward to obtain $\mathbf{R}$. Note also that the diagonal elements of $\mathbf{R}$ are the norm values of nonzero vectors, and thus $r_{i,i}$ and $r_{i+1,i+1}$ for $i = 1, 3, 5, 5, 7$ will never be zeros. The ML problem is now simpler and rather than minimizing $||\mathbf{Y} - \mathbf{CH}||_F^2$, the solution is obtained by minimizing the metric $||\bar{\mathbf{y}} - \mathbf{Rx}||_2^2$ in a layered fashion over all different combinations of the vector $\mathbf{x}$. To make this clearer, let the square $L$-QAM alphabet be given as $\mathbf{\Omega}^2$, where $\mathbf{\Omega} = \{-\sqrt{L} + 1, -\sqrt{L} + 3, \ldots, \sqrt{L} - 1\}$. Then

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x} \in \mathbf{\Omega}^8} ||\bar{\mathbf{y}} - \mathbf{Rx}||_2^2. \qquad (11)$$

Thus, $x_i$ and $x_{i+1}$ are given by

$$\arg\min_{x_i \in \Omega, x_{i+1} \in \Omega} \{|\bar{y}_{i+1} - r_{i+1,i+1}x_{i+1}|^2 + |\bar{y}_i - r_{i,i}x_i - r_{i,i+1}x_{i+1}|^2\} \qquad (12)$$

for $i = 1, 3, 5, 7$. In other words, the ML solution is obtained by jointly decoding two real symbols through a simpler $2 \times 2$ real-valued upper triangular equivalent channel matrix. Note that this simplification is obtained through the orthogonality properties in (8) and the QR decomposition in (9), resulting in (10). This is similar to but simpler than [2] and [5] in that all have joint detection of two real symbols ($N/2$ times in [2], [5] and $n_s$ times in this work) but with minimizing a norm of size $2N$ in [2] and [5] while minimizing a norm of size 2 in this work. This means that the original complex ML problem is decomposed into $n_s = 4$ parallel real-valued upper triangular problems, each of dimension 2. Writing this in matrix form, the ML solution is obtained by carrying out

$$\arg\min_{x_i \in \Omega, x_{i+1} \in \Omega} \left|\left|\begin{bmatrix} \bar{y}_i \\ \bar{y}_{i+1} \end{bmatrix} - \begin{bmatrix} r_{i,i} & r_{i,i+1} \\ 0 & r_{i+1,i+1} \end{bmatrix} \begin{bmatrix} x_i \\ x_{i+1} \end{bmatrix}\right|\right|_2^2 \qquad (13)$$

for $i = 1, 3, 5, 7$.

Obviously, this approach allows finding the ML solution faster, and requires a small number of computational operations compared to conventional ML, [2] and [5], as will be shown in Section V.

## IV. QOSTBC WITH FULL DIVERSITY

In general, QOSTBCs do not achieve the full diversity provided by the channel. In order to achieve full diversity and improve the performance at high SNR, a conventional approach in the literature suggests that half of the symbols in a quasi-orthogonal design are chosen from a signal constellation set $\mathcal{A}$ and the other half are chosen from a rotated constellation set $e^{j\phi}\mathcal{A}$ [10], [11]. Another approach is to apply multi-dimensional rotated constellations which exhibit full diversity and maximum coding gain [5], [12], [13]. However, no proper expressions for the rotation matrix of sizes greater than four exist. Applying the first approach to our algorithm introduces interference among the real symbols. Analytically, this means that the orthogonality properties defined in (8) are no longer valid, and the matrix $\mathbf{R}$ is not block diagonal any more. Instead, a two-dimensional rotation of the real symbols $(x_1, x_2, \ldots, x_{2n_s})$ that are not orthogonal to each other is always applied, thus maintaining the orthogonality properties

among the channel columns and maximizing the diversity. By this, we overcome the problem of finding proper expressions for the rotation matrix and maintain the orthogonality properties defined in (8). Moreover, this technique is applicable for an arbitrary number of antennas and is compatible with the proposed decoding algorithm.

The two-dimensional rotation matrix is defined by [5], [13] as

$$\mathbf{G} = \left[ \begin{array}{cc} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{array} \right] \qquad (14)$$

where the optimal angle is obtained by $\theta = \frac{1}{2}\mathrm{atan}(\frac{1}{2})$ for square QAM constellations [2]. In the following, we discuss the application of this rotation to QOSTBCs for $N = 4$.

Using the properties defined in (8), we combine every two real symbols that are jointly decoded into one group, then we apply a two-dimensional rotation using the matrix $\mathbf{G}$. Full diversity is achieved by replacing $s_1, s_2, s_3, s_4$ defined in [5] by $\tilde{s}_1, \tilde{s}_2, \tilde{s}_3, \tilde{s}_4$ [10], [13], where $\tilde{s}_1 = \tilde{x}_1 + j\tilde{x}_3, \tilde{s}_2 = \tilde{x}_2 + j\tilde{x}_4, \tilde{s}_3 = \tilde{x}_5 + j\tilde{x}_7, \tilde{s}_4 = \tilde{x}_6 + j\tilde{x}_8$, and the real symbols $\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_8$ are obtained by

$$\left[ \begin{array}{c} \tilde{x}_1 \\ \tilde{x}_2 \end{array} \right] = \mathbf{G} \left[ \begin{array}{c} x_1 \\ x_2 \end{array} \right], \qquad \left[ \begin{array}{c} \tilde{x}_3 \\ \tilde{x}_4 \end{array} \right] = \mathbf{G} \left[ \begin{array}{c} x_3 \\ x_4 \end{array} \right],$$

$$\left[ \begin{array}{c} \tilde{x}_5 \\ \tilde{x}_6 \end{array} \right] = \mathbf{G} \left[ \begin{array}{c} x_5 \\ x_6 \end{array} \right], \qquad \left[ \begin{array}{c} \tilde{x}_7 \\ \tilde{x}_8 \end{array} \right] = \mathbf{G} \left[ \begin{array}{c} x_7 \\ x_8 \end{array} \right] \qquad (15)$$

(recall that $\mathbf{x} = [ \begin{array}{cccc} x_1 & x_2 & \ldots & x_8 \end{array} ]$ is previously defined as $\mathbf{x} = [ \begin{array}{cccc} \breve{x}_1 & \breve{x}_3 & \ldots & \breve{x}_6 & \breve{x}_8 \end{array} ]$, and that the vector $\tilde{\mathbf{x}}$ is the rotation of $\mathbf{x}$). Incorporating the rotation matrix $\mathbf{G}$ into the system will change the equivalent channel matrix $\tilde{\mathbf{H}}$, but will keep the orthogonality properties defined in (8) unchanged. The equivalent channel matrix $\tilde{\mathbf{H}}$ after the rotation is given by

$$\left[ \begin{array}{ccc} h_1\cos\theta + h_7\sin\theta & \cdots & -h_6\cos\theta + h_4\sin\theta \\ h_2\cos\theta + h_8\sin\theta & \cdots & h_5\cos\theta - h_3\sin\theta \\ \vdots & \ddots & \vdots \\ h_8\cos\theta + h_2\sin\theta & \cdots & -h_3\cos\theta + h_5\sin\theta \end{array} \right]. \qquad (16)$$

Applying QR decomposition to $\tilde{\mathbf{H}}$ produces a block diagonal matrix $\mathbf{R}$ of the form previously discussed while maintaining the orthogonal properties defined in (8). Thus, ML detection of $n_s = 4$ parallel $2 \times 2$ real-valued sub-systems becomes possible and achieves full diversity by properly choosing the optimal rotation angle $\theta$. The proof that $\theta = \frac{1}{2}\mathrm{atan}(\frac{1}{2})$ is the optimal angle that achieves full diversity for all square QAM constellations can be found in [1].

Finally, we emphasize the fact that the proposed decoding algorithm is generic and has the same computational complexity for rotated and non-rotated QOSTBCs.

## V. COMPUTATIONAL COMPLEXITY

In this section, we compare the computational complexity of the proposed algorithm with that of conventional ML detection and the complexity of other reported algorithms in the literature. Analogous to [5], the overall complexity is measured in terms of the number of operations required to decode the transmitted signals for each block period $T$. Using the same notation as in [5], a complex multiplication is equivalent to 4 real multiplications $C_M$ and 2 real additions $C_A$, while a

complex addition is equivalent to 2 real additions. We split the complexity formula into two parts in order to represent $C_M$ and $C_A$ independently. We denote the complexity of the proposed algorithm by $\mathbf{C}_{PR}$, and show it as a two-dimensional vector where the first dimension is the number of real multiplications and the second, the number of real additions, then

$$\mathbf{C}_{PR} = n_s L(6C_M, 8C_A). \qquad (17)$$

For $N = 4$, besides the complexity in (17) the proposed algorithm requires $144C_M$ and $88C_A$ for performing QR decomposition of the channel matrix. Additionally, the computation of $\bar{\mathbf{y}} = \mathbf{Q}^H \tilde{\mathbf{y}}$ requires $64C_M$ and $56C_A$. It is important to emphasize the fact that these numbers and (17) hold for rotated and non-rotated constellations, and that all comparisons given consider the complexity of QR and the computation of $\bar{\mathbf{y}} = \mathbf{Q}^H \tilde{\mathbf{y}}$.

Conventional ML detection [4], on the other hand, performs pairwise complex symbol detection and the complexity $\mathbf{C}_{ML}$ is presented in [5] as (in the same notation as (17))

$$\mathbf{C}_{ML} = 2L^{\frac{N}{2}}\{(2N^2 + 4N)C_M, (2N^2 + 3N - 1)C_A\}. \qquad (18)$$

Obviously, $\mathbf{C}_{ML}$ increases exponentially with $N$ and polynomially with $L$, whereas $\mathbf{C}_{PR}$ is linear with $n_s$ (or equivalently $N$) and $L$. Recall that the complexity gain obtained by $\mathbf{C}_{PR}$ is for free, since this algorithm gives the optimal ML solution. The decoding algorithm presented in [5] was able to achieve ML performance while reducing the complexity from $\mathcal{O}(L^{N/2})$ to $\mathcal{O}(L^{N/4})$ compared to conventional ML detection. The complexity is

$$\mathbf{C}_{[5]} = 4L^{\frac{N}{4}}\{(N^2 + 4N)C_M, (N^2 + 3N - 1)C_A\}. \qquad (19)$$

This complexity is exponential in $N$ and polynomial in $L$, thus $\mathbf{C}_{PR}$ is more desirable. For example, for $N = 4$ and 16-QAM modulation scheme, $\mathbf{C}_{PR}$ achieves $> 71\%$ reduction in the complexity compared to $\mathbf{C}_{[5]}$, and $> 97\%$ compared to $\mathbf{C}_{ML}$. This complexity gain becomes greater as $L$ or $N$ is larger.

SD was proposed in [7] to decode QOSTBCs. The computational complexity is reduced from $\mathcal{O}((n_s)^6)$ for ML to $\mathcal{O}(2(n_s/2)^6)$ using SD, where $n_s$ is the number of transmitted symbols. However, it is also sensitive to the choice of the initial radius which is used to start the algorithm [14], [15]. The choice of the radius in [7] is based on the Gram matrix eigenvalues, which adds a computational complexity of $\mathcal{O}((2M)^3)$. Moreover, when no point is found inside the sphere, the radius is increased and the algorithm restarts, which additionally increases the computational complexity.

In [16], the decoding algorithm is based on QR decomposition of the complex channel matrix. It is only applicable to QOSTBCs with $N = 4$. It requires a number of complex computations for finding the partial metrics of the complex transmitted symbols [16]. Recall that a complex multiplication is equivalent to 4 real multiplications and 2 real additions. Furthermore, the complexity is not only a function of the number of signal points within the transmission constellation, but also a function of the sorting criterion adopted there. Unfortunately, no complexity formula was presented. However, a complexity reduction of $82\%$ compared to the conventional ML for 16-
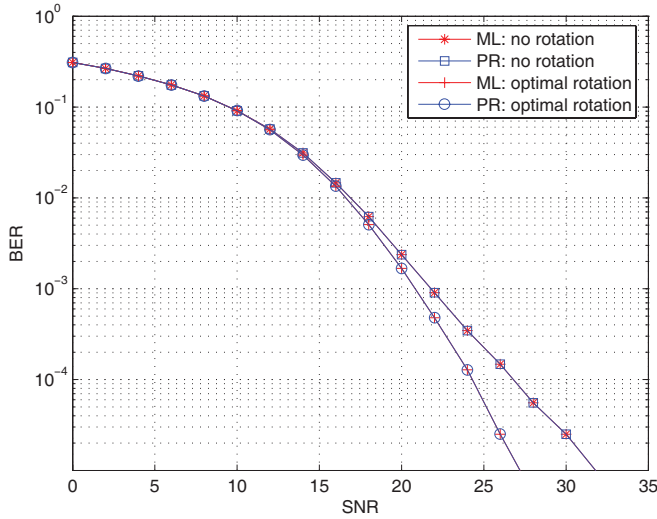
Fig. 1. BER vs SNR for the optimal proposed algorithm PR and conventional ML for space-time block codes at 4 bits/s/Hz; 4 transmit and 1 receive antennas, 16-QAM.

TABLE I
NUMBER OF REAL MULTIPLICATIONS AND REAL ADDITIONS VS $L$ FOR $N = 4$

|  | $L$ | 4 | 16 | 64 | 256 |
|---|---|---|---|---|---|
| | ML | 1536 | 24576 | 393216 | 6291456 |
| $C_M$ | [2], [5] | 512 | 2048 | 8192 | 32768 |
| | PR | 304 | 592 | 1744 | 6352 |
| | ML | 1376 | 22016 | 352256 | 5636096 |
| $C_A$ | [2], [5] | 432 | 1728 | 6912 | 27648 |
| | PR | 272 | 656 | 2192 | 8336 |

## VII. CONCLUSIONS

An efficient ML decoding algorithm based on QR decomposition was proposed for quasi-orthogonal space-time codes. The proposed algorithm was shown to achieve full diversity by applying a two-dimensional constellation rotation. The performance was shown to be optimal while reducing the decoding complexity significantly compared to conventional ML and best algorithms in the literature.

QAM was reported. Thus, taking the conventional ML as a baseline, we obtain a reduction of $> 97\%$ for the same $L$ and $N$.

Finally, the decoding algorithm presented in [2] has a complexity which is comparable to the complexity of the algorithm proposed in [5] which is a function of $\mathcal{O}(L^{N/4})$, since both algorithms perform joint detection of two real symbols. Therefore, the same complexity gain of $\mathbf{C}_{PR}$ over $\mathbf{C}_{[5]}$ is applicable also over $\mathbf{C}_{[2]}$. However, the algorithm of [2] suffers a performance loss of $0.4$ dB compared to ML.

## VI. SIMULATION RESULTS

We provide simulation results for the proposed algorithm for $N = 8$ and $N = 4$. We denote the proposed algorithm by PR. In all simulations, we consider one receive antenna. Moreover, we use the same rotating matrix $\mathbf{G}$ defined in (14), and the same rotation angle $\theta$ given by $\theta = \frac{1}{2}\text{atan}(\frac{1}{2})$ in order to achieve full transmit diversity.

Figure 1 provides simulation results for the transmission of 4 bits/s/Hz with $N = 4$ and 16-QAM. We compare PR with conventional ML detection with and without rotation. PR achieves full diversity through the rotation matrix $\mathbf{G}$, while ML detection achieves it by replacing the transmitted symbols $s_3, s_4$ with $s_3 e^{j\pi/4}, s_4 e^{j\pi/4}$ (i.e., choosing half of the transmitted symbols from a rotated constellation set $e^{j\phi}\mathcal{A}$ as discussed earlier) [3]. PR achieves the same performance as conventional ML, with a substantial reduction in the decoding complexity. Using (17) and (19), we find that this complexity reduction is $> 97\%$.

The complexity is measured in terms of the number of real multiplications required to decode one block of transmitted symbols as a function of the constellation size $L$. The complexity of [2] is in the order of $\mathcal{O}(L^{N/4})$ which is equal to $\mathbf{C}_{[5]}$ and will be denoted $\mathbf{C}_{[2],[5]}$ in the sequel. In Table I, we give a comparison between $\mathbf{C}_{ML}$, $\mathbf{C}_{[2],[5]}$, and $\mathbf{C}_{PR}$ in terms of the number of real multiplication and real additions considering $N = 4$ for different constellation sizes. The number of multiplications and additions shown include the computation of QR, and $\bar{\mathbf{y}} = \mathbf{Q}^H \tilde{\mathbf{y}}$.

## REFERENCES

[1] L. Azzam and E. Ayanoglu, "Maximum likelihood detection of quasi-orthogonal space-time block codes: analysis and simplification," to appear in *IEEE ICC*, May 2008.
[2] C. Yuen, Y. Guan, and T. Tjhung, "Quasi-orthogonal STBC with minimum decoding complexity," *IEEE Trans. Wireless Commun.*, vol. 4, no. 5, pp. 2089–2094, Sept. 2005.
[3] H. Jafarkhani, *Space-Time Coding: Theory and Practice*. Cambridge University Press, 2005.
[4] ——, "A quasi-orthogonal space-time block code," *IEEE Trans. Commun.*, vol. 49, no. 1, pp. 1–4, Jan. 2001.
[5] H. Lee, J.Cho, J. Kim, and I. Lee, "An efficient decoding algorithm for STBC with multi-dimensional rotated constellations," in *Proc. IEEE ICC*, vol. 12, pp. 5558–5563, June 2006.
[6] C. Yuen, Y. Guan, and T. Tjhung, "Full-rate full-diversity STBC with constellation rotation," in *Proc. IEEE VTC*, vol. 1, pp. 296–300, Apr. 2003.
[7] A. Peng, Kim, and S. Yousefi, "Low-complexity sphere decoding algorithm for quasi-orthogonal space-time block codes," *IEEE Trans. Commun.*, vol. 54, no. 3, pp. 377–382, Mar. 2006.
[8] L. Azzam and E. Ayanoglu, "Reduced complexity sphere decoding for square QAM via a new lattice representation," in *Proc. IEEE GLOBECOM*, pp. 4242–4246, Nov. 2007.
[9] H. Gamal and M. Damen, "Universal space-time coding," *IEEE Trans. Inform. Theory*, vol. 49, pp. 1097–1119, June 2003.
[10] N. Sharma and C. Papadias, "Improved quasi-orthogonal codes through constellation rotation," *IEEE Trans. Commun.*, vol. 51, no. 3, pp. 332–335, Mar. 2003.
[11] O. Tirkkonen, "Optimizing space-time block codes by constellation rotations," in *Proc. Finnish Wireless Communications Workshop (FWWC)*, pp. 59–60, Oct. 2001.
[12] J. Boutros, E. Viterbo, C. Rastello, and J. Belfiore, "Good lattice constellation for both Rayleigh fading and Gaussian channels," *IEEE Trans. Inform. Theory*, vol. 42, pp. 502–518, Mar. 1996.
[13] J. Boutros and E. Viterbo, "Signal space diversity: a power- and bandwidth-efficient diversity technique for the Rayleigh fading channel," *IEEE Trans. Inform. Theory*, vol. 44, pp. 1453–1467, July 1998.
[14] W. Zhao and G. Giannakis, "Sphere decoding algorithms with improved radius search," *IEEE Trans. Commun.*, vol. 53, pp. 1104–1109, July 2005.
[15] B. Hassibi and H. Vikalo, "On the sphere-decoding algorithm–I: expected complexity," *IEEE Trans. Signal Processing*, vol. 53, no. 8, pp. 2806–2818, Aug. 2005.
[16] M. Le, V. Pham, L. Mai, and G. Yoon, "Low-complexity maximum-likelihood decoder for four-transmit-antenna quasi-othogonal space-time block code," *IEEE Trans. Commun.*, vol. 53, no. 11, pp. 1817–1821, Nov. 2005.