An Equalizer Design Technique for the PCM Modem: A New Modem for the Digital Public Switched Network

Ender Ayanoglu, *Fellow, IEEE*, Nuri R. Dagdeviren, Glenn D. Golden, *Member, IEEE*, and James E. Mazo, *Fellow, IEEE*

***Abstract*—Modems designed for the public switched telephone network (PSTN) have conventionally been based on modeling assumptions which view the PSTN connection as an essentially analog medium. However, as the PSTN evolves toward all digital transport and switching, and particularly as major traffic sources, such as Internet service providers, increasingly have direct digital connections to the PSTN, it is appropriate to revisit the model assumptions. Recently, several modem and chipset manufacturers have announced "56K" modems based on an emerging system paradigm in which one user (a residential Internet subscriber) has an analog connection to the PSTN, and the other (an Internet service provider) has a digital one. ITU-T is expected to finalize details of a corresponding recommendation in 1998. With this configuration, modem designs based on signaling with the μ-law alphabet become feasible, and the conventional Shannon limit disappears as the quantization noise is avoided. Thus, the conventional Shannon limit of about 36 kb/s can be beaten, and it is possible to approach the digital transmission rate of 64 kb/s. Modems employing this general approach have become known as μ-law or pulse-code modulation (PCM) modems. In this paper we present a signaling technique and the sampling theory based on this technique, display the structure and operating principles of a PCM modem equalizer, and show how this equalizer problem can be cast in the language of multiinput–multioutput (MIMO) system theory.**

***Index Terms*—56K modem, equalization, multiinput–multioutput (MIMO) systems, nonuniform sampling, Nyquist bandwidth limit, PCM modem, Shannon capacity limit, V.90.**

I. Introduction

To BEGIN, we briefly review the basic elements of end-to-end transmission within the public switched telephone network (PSTN) and discuss the basic limits to data communication over this channel. We then describe how these limits can be mitigated under the new hybrid analog/digital (A/D) system paradigm, subject to the solution of some challenging theoretical and practical problems. We will address solutions to these challenges in later sections.

Fig. 1(a) shows the three tandemed components of an end-user to end-user analog telephone connection through the U.S. domestic PSTN.[1] In the figure, sections A and C of the connection are conventional twisted pairs ("analog subscriber loops" or "analog loops") which transport the *analog* signals from telephone sets of each user to their associated local central offices. At the central office, the analog signals are converted to 64-kb/s DS0 digital data streams by a channel unit[2] filter and codec, which together implement a band-limiting filter followed by a nonlinear encoding rule and subsequent A/D conversion. The resulting DS0 streams are transported to their respective destination central offices via digital transmission facilities B.

Consider only one direction of transmission in Fig. 1(a), say, from user 1 to user 2. At the central office, user 1's loop signal is first bandlimited using a filter whose frequency response is specified by [1], and is typically as shown in [2]. The band-limited analog signal is then sampled at a rate of 8 ksamples/s, and then converted into an 8-b digital representation using a nonlinear mapping rule referred to as μ-law encoding. This encoding is approximately logarithmic and its purpose is to permit relatively large dynamic range voice signals to be represented with only 8 b/sample. The μ-law rule was designed specifically so that the distortion (i.e., quantization noise) resulting from the mapping of analog voltage samples into the nearest one of the 255 μ-law quantization levels is relatively constant and not perceptually objectionable for voice transmission.

However, users 1 and 2 may choose to use modems to transmit digital data over the configuration in Fig. 1(a). Using conventional modems, the users' digital data is encoded into a symbol sequence, which is in turn represented as an appropriately band-limited analog signal, which can be transmitted over the approximately 3.5-kHz bandwidth available on the end-to-end connection. A matching modem at the far end operates on the received signal and estimates the transmitted symbol sequence.

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E. Ayanoglu and G. D. Golden are with Bell Laboratories, Lucent Technologies, Holmdel, NJ 07733-3030 USA.

N. R. Dagdeviren is with Bell Laboratories, Lucent Technologies, Middletown, NJ 07748-4801 USA.

J. E. Mazo is with Bell Laboratories, Lucent Technologies, Murray Hill, NJ 07974-0636 USA.

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[1] For consistency, throughout the paper we will assume the domestic U.S. PSTN, in which the so called μ-law quantization rule is used. The analysis and results are general, though, and apply with straightforward modification to any PSTN, e.g., the European A-law based system.

[2] We will refer to a generic channel unit to describe the network element that performs the A/D and D/A conversion. This unit can, in reality, be in a line card on a digital switch or a digital subscriber loop carrier, or a channel unit in a channel bank or other equipment. Their behavior is standard for reasons of interoperability; therefore, they all behave identically for our purposes.

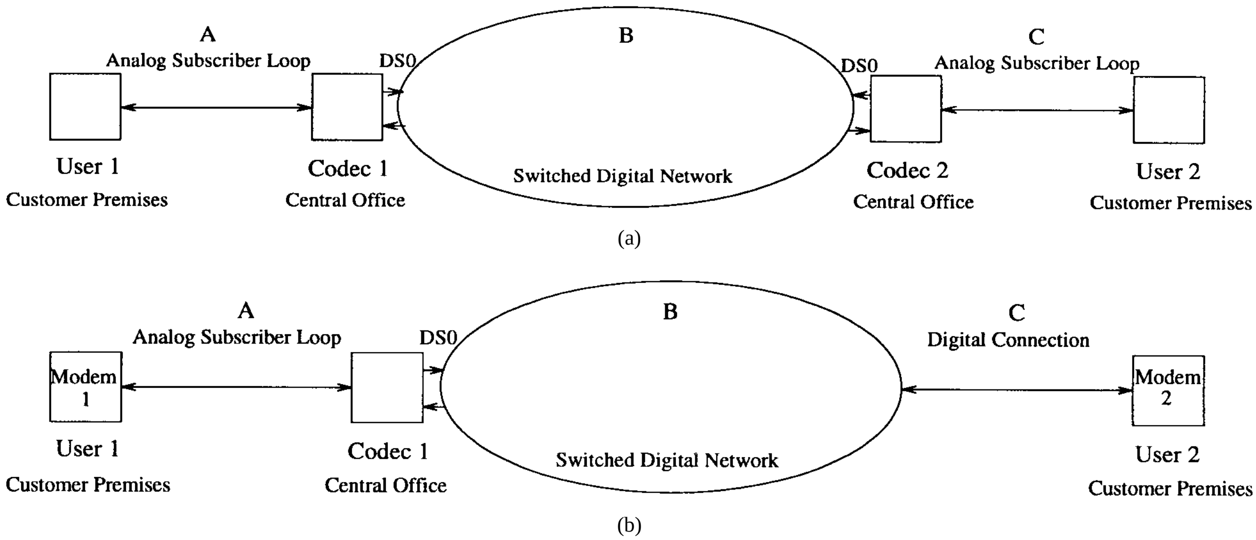

Fig. 1. Connections through a PSTN. (a) Analog end-to-end connection. (b) Hybrid A/D connection.

Conventional modems do not explicitly take into account the A/D conversion and μ-law encoding processes in the telephone transmission system, and so the analog signals generated by conventional modems suffer the same quantization distortion effects as do voice signals. In fact, when PSTN internal transport is all digital, the primary source of symbol estimation error is μ-law quantization noise. The theoretical transmission rate limit for this channel is traditionally based on modeling the quantization noise as additive white Gaussian noise (AWGN) and then invoking the Shannon capacity formula [3]

$$C = W \log_2(1 + \mathrm{SNR}) \text{ b/s} \tag{1}$$

for a channel strictly bandlimited to W Hz and having a signal-to-noise ratio (SNR). Depending on loop characteristics, which vary widely, the effective SNR due to quantization noise is typically in the range of 33–39 dB and, assuming a bandwidth of approximately 3–3.5 kHz, evaluation of (1) results in a channel capacity of $C \approx 33$–45 kb/s.

The most recent standard conventional modem design based on these assumptions is the ITU-T Standard V.34, which can achieve 28.8 kb/s over an appreciable fraction of the U.S. subscriber loop ensemble. With some additional techniques, this can be increased to about 33.6 kb/s on very good connections.

In the absence of quantization noise, assuming no other significant impairments are present, and if the channel is strictly bandlimited, the highest transmission rate for the system of Fig. 1(a) would no longer be imposed by the Shannon limit but by the combination of Nyquist bandwidth limitation and the 8-b resolution of pulse-code modulation (PCM) μ-law encoding. Nyquist theory [4] allows that at most $2W$ independent symbols/s can be transmitted over a channel strictly bandlimited to W Hz. Since each sample is quantized to 8 b, the Nyquist-limited capacity becomes

$$C = 2W \text{ samples/s} = 16W \text{ b/s} \tag{2}$$

or about 48–64 kb/s for $W = 3$–4 kHz.

The significance of this observation [5] and the practicality of implementation using an analog $\leftrightarrow$ digital configuration was recognized early on [6]–[8]. A prototype modem using the PCM approach in an analog $\leftrightarrow$ digital configuration was developed and demonstrated at Bell Laboratories in 1993. Since that time, modems based on this general approach have come to be known as μ-law or PCM modems. A modulation and equalization technique based on this methodology is described in [5], [6], and [8]–[10]. In this paper we build upon this work here to establish general relationships between bandwidth, rate, and sampling schedule. We also analyze and present operating principles for this modem equalizer structure, and establish some interesting relationships between this problem and multiinput–multioutput (MIMO) system theory.

II. PCM Modems: Overview

The central idea behind the PCM modem is to avoid the effects of quantization distortion by utilizing the μ-law quantization levels themselves as the channel symbol alphabet. In the upstream direction the analog loop is appropriately equalized at the transmitting modem and the transmit timing is precisely adjusted so that the analog voltage sampled by the codec passes through the desired quantization levels precisely at its 8-kHz sampling instants. The desired result is that the users' transmitted symbol sequence is explicitly transported across the network in digital form. In the downstream direction the reverse occurs—the loop is equalized at the receiver and the receiver sample timing is adjusted so as to again result in voltage samples that are just those quantization levels impressed by the codec.

However, the accomplishment of this task for the end-to-end configuration of Fig. 1(a) is difficult, and perhaps impossible, primarily because of the presence of two tandemed codecs and the absence of any side information from within the network which could be used for training the equalizers and echo cancelers. (This issue will be elaborated on in Section VI.) On the other hand, a hybrid A/D configuration, shown in Fig. 1(b), has become increasingly common as residential users access the Internet via service providers who can economically employ direct digital connections to the PSTN. Although sections A and B remain the same as in Fig. 1(a), section C of the connection in Fig. 1(b) is now an all-digital transport of the DS0 streams, typically via T1 or ISDN, to and from the network at large. Thus, there is only a single codec present in this configuration, located at the channel unit termination of section A. Also, user 2's modem is, in effect, inside the network and so can participate directly in the equalizer training process, e.g., by sending a known training sequence. With this system configuration, the PCM modem approach becomes far less complicated.

In fact, with this hybrid configuration and the assumption of downstream-only transmission, the primary remaining technical impediment is accurate equalization of the loop. However, despite superficial similarities, it is not obvious how to go about adapting conventional equalization techniques to this problem—the PCM modem equalization problem is inherently different than that for the conventional modem because we are constrained by the network hardware to employ a particular symbol set and, as will be seen, we must also conform to symbol sampling times which are neither uniform nor under the modem designers' control. In what follows we display some of the fundamental relationships between available bandwidth, channel sampling schedule, and the design of equalizers for this modem.

In the next section, for the purpose of developing the basic structure for the PCM modem equalizer, we assume that the channel unit filter at the central office is strictly bandlimited. On real loops, the filters roll off gradually and spectrum outside the nominal passband can be exploited. We discuss this issue in Section VII.

III. Signal Generation for the Ideal Channel

Basic Nyquist theory provides that within a one-sided bandwidth of W Hz it is possible to transmit independent symbols T_s s apart as long as the symbol rate $f_s = 1/T_s \leq 2W$. However, it is known [11], and we show by construction below that it is also possible to transmit information within a bandwidth of W Hz while employing a symbol interval of $T_s = 1/f_s$ with $f_s > 2W$, as long as the actual information rate is limited to $2W$ symbols/s. This can be accomplished in principle by permitting only $2W$ symbols/s to be independently chosen, with the extra $f_s - 2W$ symbols/s carrying limited or no new information or, more precisely, information which is in part or fully determined by the independent $2W$ symbols/s [5], [6]. A simple way to construct such signals is to permit only some subset M out of every N consecutive symbols to be independently chosen, while the remaining $N - M$ become determined by the bandwidth restriction. In the next section we will be more precise about this, deriving a general relation between f_s, W, M, N, and the available spectrum. For the time being, we are interested in showing only the existence of such signals for the case of interest. We will construct such signals first for an ideal low-pass channel bandlimited to 3.5 kHz and then for the more relevant case of a bandpass channel, which, for illustrative purposes, we choose to be strictly bandlimited to 500 Hz–3.5 kHz [6].

First, we will concentrate on the ideal (flat magnitude, linear phase) low-pass channel strictly bandlimited to $W = 3.5$ kHz. We choose $M = 7$, $f_s = 8$ kHz, and groups of $N = 8$ samples. Thus, the first seven samples of each group are independent information-bearing symbols, and every eighth sample is determined by the first seven independent samples. The average symbol rate of such a signal is $2W = 7$ ksymbols/s, although it is composed of samples generated at $f_s = 8$ kHz.

Such a signal can be written as [5], [6]

$$s(t) = \sum_{n=-\infty}^{\infty} \sum_{p=1}^{7} a_{p,n} y_p(t - 8nT_s) \tag{3}$$

where

$$\begin{aligned} t_p &= (p-1)T_s, \qquad p = 1, 2, \cdots, 7 \\ y_p(t) &= \operatorname{sinc}\left(\frac{t-t_p}{8T_s}\right) \prod_{q=1,\, q\neq p}^{7} \sin \frac{\pi(t-t_q)}{8T_s}, \\ & \qquad p = 1, 2, \cdots, 7 \\ \operatorname{sinc}(x) &= \frac{\sin(\pi x)}{\pi x} \end{aligned} \tag{4}$$

and $a_{p,n}$ is the pth data-bearing sample from the nth group of eight samples. Thus, $s(t)$ is built from the M component pulses $y_p(t)$. Two typical pulses $y_1(t)$ and $y_2(t)$ are shown in Fig. 2(a) and (b), respectively. Equation (3) can be interpreted as the multiplexing of M different pulse-amplitude modulation (PAM) streams, with the basic pulse in the pth PAM stream being $y_p(t)$. The product of sinusoids in $y_p(t)$ ensures desired zero crossings and the bandwidth constraint, as will become clearer in the sequel.

First, observe that the prescribed low-pass bandwidth constraint is met—from (4), in the frequency domain, each $y_p(t)$ consists of a rectangular function bandlimited to 500 Hz, modulated by a different 500-Hz sine function six times. Hence, $y_p(t)$ is bandlimited to 3.5 kHz. By linearity, $s(t)$ in (3) is also bandlimited to 3.5 kHz.

Second, observe that the $a_{p,n}$ can be recovered by appropriate sampling of the signal at the original sampling rate f_s. Consider $y_1(t)$, shown in Fig. 2(a). The pulse has a nonzero value at $t = 0$, the time at which the underlying data symbol (i.e., the pulse height) can be recovered. There is no intersymbol interference (ISI) at integer multiples of T_s, except for $t/T_s = \cdots, -9, -1, 7, 15, \cdots = -1 + k8$ for $-\infty \leq k \leq \infty$, or t/T_s congruent to $7 \bmod 8$, and no pulse is inserted at $t/T_s = 7 \bmod 8$. The pulse $y_2(t)$, shown in Fig. 2(b), is similar to $y_1(t)$ except that the time at which data is inserted is $t = T_s$. Continuing in this manner, if the pulse train (3) is used to encode every $M = 7$ out of $N = 8$ samples to transmit data $M/N = 2W/f_s$, then no ISI results. Also note that if, say, α additional hertz of bandwidth is available, the pulses $y_p(t)$ can be constructed to decay at a rate faster than $1/t$ if the sinc factor in (4) is replaced by a Nyquist pulse with rolloff factor α, while still preserving the zero-ISI property at the 7-out-of-8 symbol sampling times.

For another ideal channel, bandlimited to 500 Hz–3.5 kHz, we can construct a similar signal by employing $M = 6$ basic

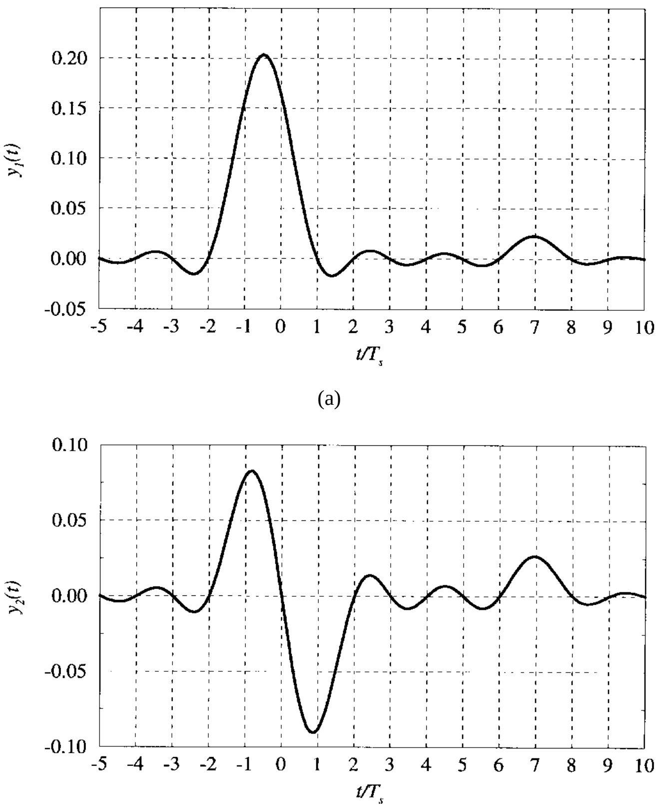

Fig. 2. Basic pulses (a) $y_1(t)$ and (b) $y_2(t)$ for the low-pass 3.5-kHz band-limited channel.

pulse waveforms $y_p(t)$ as follows:

$$s(t) = \sum_{n=-\infty}^{\infty} \sum_{p=1}^{6} a_{p,\,n} y_p(t - 8nT_s) \tag{5}$$

where

$$t_p = (p-1)T_s, \qquad p = 1, 2, \cdots, 6$$

$$y_p(t) = \mathrm{sinc}\left(\frac{t-t_p}{8T_s}\right) \cos \frac{\pi t}{2T_s} \prod_{q=1,\, q\neq p,\, q \text{ odd}}^{5} \sin \frac{\pi(t-t_q)}{8T_s} \tag{6}$$

for $p = 1, 3, 5$, and

$$y_p(t) = \mathrm{sinc}\left(\frac{t-t_p}{8T_s}\right) \sin \frac{\pi t}{2T_s} \prod_{q=2,\, q\neq p,\, q \text{ even}}^{6} \sin \frac{\pi(t-t_q)}{8T_s} \tag{7}$$

for $p = 2, 4, 6$. There are now two dependent symbol/sample times, $t/T_s = 6 \bmod 8$ and $t/T_s = 7 \bmod 8$. The signals $y_1(t)$, $y_3(t)$, and $y_5(t)$ are nonzero at $t/T_s = 6 \bmod 8$, and $y_2(t)$, $y_4(t)$, and $y_6(t)$ are nonzero at $t/T_s = 7 \bmod 8$, in addition to the corresponding data-bearing times $t = (p-1)T_s$, $1 \leq p \leq 6$ for $y_p(t)$ and, hence, the signal $s(t)$ in (5) does not bear any data at $t/T_s = 6 \bmod 8$ and $t/T_s = 7 \bmod 8$. Note that the rectangular function in the frequency domain due to the sinc function is shifted to 1.5–2.5 kHz by $\cos(\pi t/2T_s)$ or $\sin(\pi t/2T_s)$, and the two sinusoids in the product form move it to 500 Hz–3.5 kHz, therefore satisfying the bandwidth restriction.

Intuitively, it might be expected that this type of construction could be extended to the generalized band-limited channel, i.e., a channel having W-Hz total one-sided spectral support, not necessarily contiguous. It might be expected that this would work for samples generated at rate $f_s > 2W$, with M samples out of every N bearing independent data, and

$$\frac{M}{N} \leq \frac{2W}{f_s}. \tag{8}$$

However, a key point is that not every M, N pair satisfying (8) will work. For example, in the bandpass case above, $M = 3$ and $N = 4$ satisfy (8), but it is not possible to find a signal set of M pulses $y_p(t)$ which will achieve the desired result. Furthermore, even if valid M and N are found, the choice of which particular M out of N to choose as data-bearing samples is not arbitrary, and again, not every choice will work. In any case, the design of such signals for the ideal channel is not in itself directly useful in the PCM modem problem since real-world channels are not flat and linear phase and equalization will invariably be necessary.

In the next section we will first state a general theory which allows us to determine both permissible M and N, as well as which particular M out of N time instances are appropriate for an arbitrary nonideal generalized strictly band-limited channel and f_s. Following that, we will describe methods to train equalizers for this modem.

IV. Equalizer Theory

Let us define $T = NT_s$. In order to establish a general equalizer theory for data transmission when the sampling frequency exceeds the Nyquist rate, it is useful to partition the pulse family $y_p(t)$ into a set of composite channel-plus-equalizer impulse responses

$$p_k(t) = h(t) * r^k(t), \qquad 1 \leq k \leq M \tag{9}$$

along the lines shown in Fig. 3(a), which represents the downstream direction of transmission, i.e., from user 2 to user 1. Maintaining the same M-out-of-N notation from the previous section, $r^k(t)$ is now interpreted as the receive filter (equalizer) for the kth data-bearing member of each group of N samples, and for now we require that the M data-bearing samples be consecutive. Note that on the network side, user 2's modem controls the symbol samples presented to the codec, and we have forced the nondata-bearing $N - M$ samples to zero.[3]

With this notation in place, the zero-forcing requirements for system of Fig. 3(a) $p_k(t)$ can be written compactly as

$$p_k[nT + (m-1)T_s] = \delta_n \delta_{k-m}, \qquad 1 \leq k,\, m \leq M \tag{10}$$

[3] Although we consider here the equalization problem for the downstream direction, the analysis applies with straightforward modification for the upstream direction as well, as shown in Fig. 3(b), by simply replacing the $r^k(t)$ with $x^k(t)$.

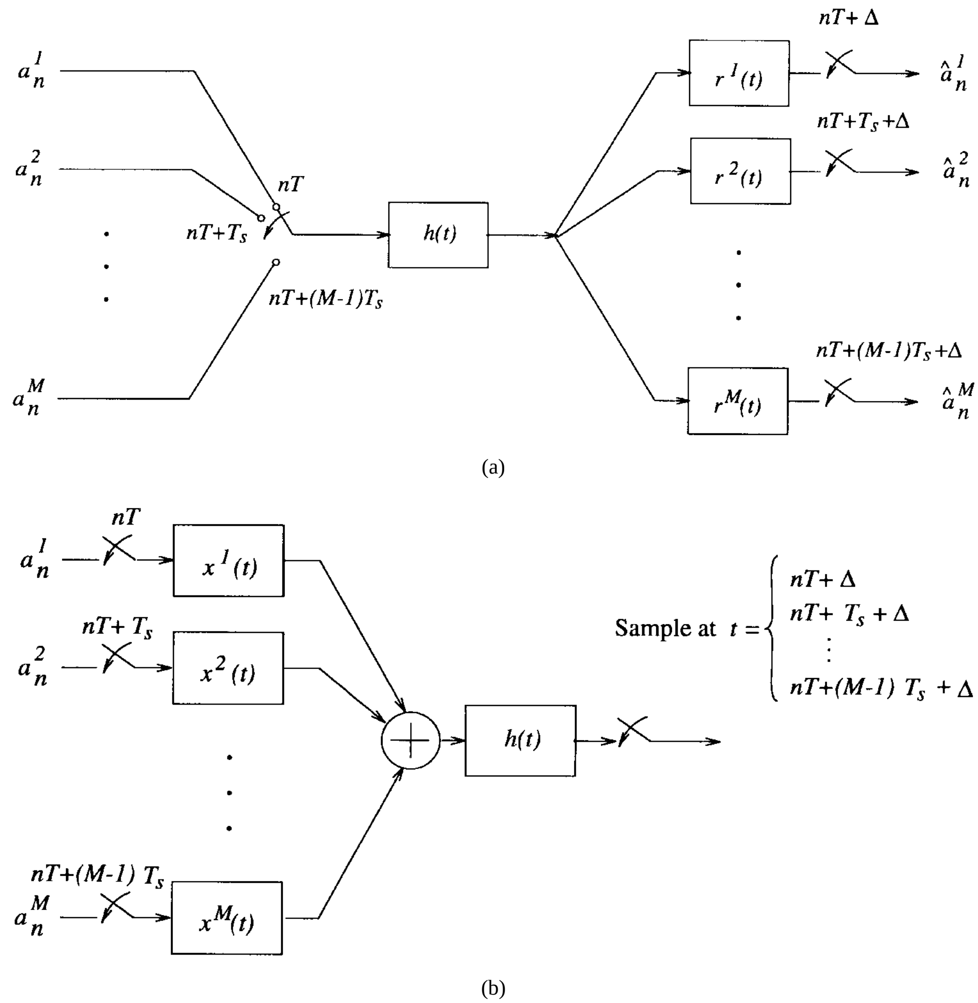

Fig. 3. Equalizer configurations for the PCM modem. (a) Downstream. (b) Upstream.

for all n. From (10)

$$\sum_{n=-\infty}^{\infty} p_k[nT+(m-1)T_s]\delta[t-nT-(m-1)T_s] = \delta_{k-m}\delta[t-(k-1)T_s] \quad (11)$$

or

$$p_k(t)\sum_{n=-\infty}^{\infty} \delta[t-nT-(m-1)T_s] = \delta_{k-m}\delta[t-(k-1)T_s]. \quad (12)$$

The Fourier transform of (12) is just

$$\frac{1}{T}\sum_{n=-\infty}^{\infty} W^{-(m-1)n}P_k\Big(f+\frac{n}{T}\Big) = \delta_{k-m}e^{-\hat{\imath}2\pi(k-1)fT_s} \quad (13)$$

where $W=e^{-\hat{\imath}(2\pi/N)}$, $\hat{\imath}=\sqrt{-1}$, $1\le k,\ m\le M$.

Noting that $W^i = W^{i+kN}$ for all i and k, (13) can be written as

$$\frac{1}{T}\sum_{n=0}^{N-1} W^{-(m-1)n}\sum_{l=-\infty}^{\infty} P_k\Big(f+\frac{n}{T}+\frac{l}{T_s}\Big) = \delta_{k-m}e^{-\hat{\imath}2\pi(k-1)fT_s} \quad (14)$$

for $1\le k,\ m\le M$, or, equivalently

$$\frac{1}{T}\begin{bmatrix} 1 & 1 & 1 & \cdots & \\ 1 & W^{-1} & W^{-2} & \cdots & W^{-(N-1)} \\ 1 & W^{-2} & W^{-4} & \cdots & W^{-2(N-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & W^{-(M-1)} & W^{-(M-1)2} & \cdots & W^{-(M-1)(N-1)} \end{bmatrix} \cdot \begin{bmatrix} \overline{P}_1^{(0)} & \overline{P}_2^{(0)} & \overline{P}_3^{(0)} & \cdots & \overline{P}_M^{(0)} \\ \overline{P}_1^{(1)} & \overline{P}_2^{(1)} & \overline{P}_3^{(1)} & \cdots & \overline{P}_M^{(1)} \\ \overline{P}_1^{(2)} & \overline{P}_2^{(2)} & \overline{P}_3^{(2)} & \cdots & \overline{P}_M^{(2)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \overline{P}_1^{(N-1)} & \overline{P}_2^{(N-1)} & \overline{P}_3^{(N-1)} & \cdots & \overline{P}_M^{(N-1)} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & e^{-\hat{\imath}2\pi fT_s} & 0 & \cdots & 0 \\ 0 & 0 & e^{-\hat{\imath}4\pi fT_s} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & e^{-\hat{\imath}2\pi(M-1)fT_s} \end{bmatrix} \quad (15)$$

where

$$\overline{P}_k^{(n)} \triangleq \sum_{l=-\infty}^{\infty} P_k\Big(f+\frac{n}{T}+\frac{l}{T_s}\Big). \quad (16)$$

Note that (15) relates frequency translates of the composite folded spectra of the pulse system $p_k(t)$, which, from (16), are seen to be folded modulo $1/T_s$ Hz. For any particular frequency $f_0 \in (-1/2T_s, 1/2T_s)$, N translates of f_0 are formed at $f_0 + n/T$, $n = 0, 1, \cdots, N-1$, the nth of these being $\overline{P}_k^{(n)}$. (Note the periodic nature of the folded spectrum.) Together, these $\overline{P}_k^{(n)}$ must satisfy the inverse DFT relation (15), which represents (10) in the frequency domain.

Let us define $\mathbf{p}_k = [\overline{P}_k^{(0)}, \overline{P}_k^{(1)}, \cdots, \overline{P}_k^{(N-1)}]^T$. For notational simplicity, we will also define $\mathbf{W} = [W^{-(i-1)(j-1)}]_{M\times N}$ and $\mathbf{e}_k$ as the unit vector in the kth direction. Then (15) can be written as

$$\frac{1}{T}\mathbf{W}\mathbf{p}_k = e^{-\hat{\imath}2\pi(k-1)fT_s}\mathbf{e}_k, \qquad 1 \le k \le M. \tag{17}$$

Note that $\mathbf{W}$ is an inverse DFT matrix. Thus, the left-hand side of (17) is an inverse DFT relation; each member of the vector on the right-hand side corresponds to the value of $p_k(t)$ at time $(m-1)T_s$, $1 \le m \le M$. These values are specified—for $m \ne k$, they are zero; for $m = k$, they are unity (within the time reference adjustment by $e^{-\hat{\imath}2\pi(k-1)fT_s}$). The values $M < m \le N$ are not specified. Thus, (17) expresses the time domain conditions of (10) in the frequency domain, but by means of an inverse DFT. It should be noted that (17) should be valid for all $|f| \le 1/2T_s$, similar to the conventional Nyquist condition. Note that any function that satisfies the zero-crossing conditions expressed in (10) will satisfy (17). Thus, the examples in (3)–(7) satisfy (17).

Note that if $H(f)$ vanishes for some values of f, then the sample-rate folded composite spectrum

$$\begin{aligned}\overline{P}_k(f) &= \sum_{l=-\infty}^{\infty} P_k\left(f + \frac{l}{T_s}\right)\\ &= \sum_{l=-\infty}^{\infty} H\left(f + \frac{l}{T_s}\right)R_k\left(f + \frac{l}{T_s}\right)\end{aligned} \tag{18}$$

may vanish as well, and thus some elements of $\mathbf{p}_k$ can become zero. Define $\tilde{\mathbf{p}}_k$ as the vector obtained by deleting the zeros from $\mathbf{p}_k$ and define $\tilde{\mathbf{W}}$ as the matrix obtained by deleting the corresponding columns of $\mathbf{W}$. Then (17) becomes

$$\frac{1}{T}\tilde{\mathbf{W}}\tilde{\mathbf{p}}_k = e^{-\hat{\imath}2\pi(k-1)fT_s}\mathbf{e}_k, \qquad 1 \le k \le M. \tag{19}$$

The matrix $\mathbf{W}$ is Fourier [12] and, therefore, rank $\tilde{\mathbf{W}} = \min(M, \dim \tilde{\mathbf{p}}_k)$. If $\dim \tilde{\mathbf{p}}_k < M$, then (19) becomes an inconsistent set of linear equations. For (19) to have *at least* one solution, we must have $\dim \tilde{\mathbf{p}}_k \ge M$ for all f. In other words, the configuration in Fig. 3(a) can support M consecutive data-bearing sample times, as described above, as long as for any $f_0 \in (-1/2T_s, 1/2T_s)$, at least M Nyquist translates lie in the support of one period of the sample-rate folded channel spectrum $\sum_{l=-\infty}^{\infty} H(f_0 + l/T_s)$.

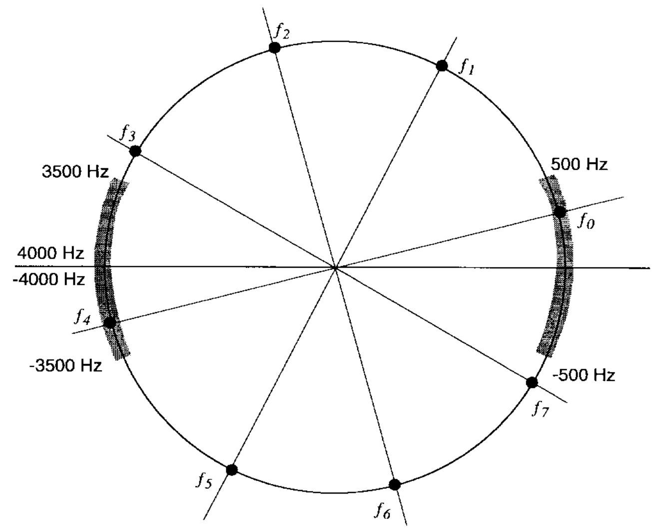

Fig. 4. Description of the spectral condition on the unit circle. For each f, there should be M nonzero Nyquist translates of the function $\overline{H}(f) = \sum_{l=-\infty}^{\infty} H(f + l/T_s)$ spaced $2\pi/N$ radians apart. Equivalently, $\overline{H}(f)$ should have M translates at $1/T$ Hz available.

This requirement can be applied to the idealized passband channel with a one-sided bandwidth of 3 kHz (500–3500 Hz). Using a sampling rate $1/T_s$ of 8 kHz and $M = 6$, $N = 8$, it can be seen that for any choice of f_0, six 1-kHz translates are always available in the support of one period of the sample-rate folded spectrum. For this channel, our earlier assertion that $M = 3$, $N = 4$ will not work can be demonstrated by examining the 2-kHz translates of, for example, $f_0 = 0$ Hz.

The above criterion can be described geometrically on the unit circle. Fig. 4 shows the support regions of the sample-rate folded spectrum $\overline{H}(f) = \sum_{l=-\infty}^{\infty} H(f + l/T_s)$ for the 500–3500 Hz passband channel plotted on the unit circle, where f takes values from $-1/2T_s$ to $1/2T_s$, corresponding to the range from $-\pi$ to π. It can be seen that for any f, at least six of the eight Nyquist translates of f lie in the support regions on the unit circle.

In the development above we assumed consecutive sampling times $1 \le m \le M$. When sampling is not consecutive, the analysis holds for (10)–(14) with the condition $1 \le m \le M$ replaced by the condition that m takes on M values from the set $\{1, 2, \cdots, N\}$. Let us denote this set of m's $\mathcal{M} \triangleq \{m_i\colon 1 \le m_i \le N \text{ for } 1 \le i \le M,\ m_i \ne m_j \text{ for } i \ne j\}$. Then, the matrix on the left-hand side of (15), $\mathbf{W} = [W^{-(i-1)(j-1)}]_{M\times N}$ is replaced by $\mathbf{W} = [W^{-(m_i-1)(j-1)}]_{M\times N}$, and the matrix on the right-hand side of (15) $[\delta_{i-j}e^{-\hat{\imath}2\pi(i-1)fT_s}]_{M\times M}$ is replaced with $[\delta_{i-j}e^{-\hat{\imath}2\pi(m_i-1)fT_s}]_{M\times M}$. Thus, in this case, the nonsingularity of $M \times M$ submatrices of $\mathbf{W} = [W^{-(m_i-1)(j-1)}]_{M\times N}$ become a condition for realizability *in addition* to the spectral condition mentioned above. For example, for $\mathcal{M} = \{1, 2, 3, 5, 6, 7\}$ in the $M = 6$, $N = 8$, $1/T = 1$-kHz passband problem above, the submatrix corresponding to columns 1–3 and 5–7 of $\mathbf{W}$ becomes singular at, for example, $f_0 = 0$ Hz. It is known that when N is prime, all such submatrices are nonsingular [13]. Thus, when N is prime, any set of M sampling times will do, as long as, for all f, at least M nonzero $1/T$-spaced translates are available in the sample-rate folded spectrum.

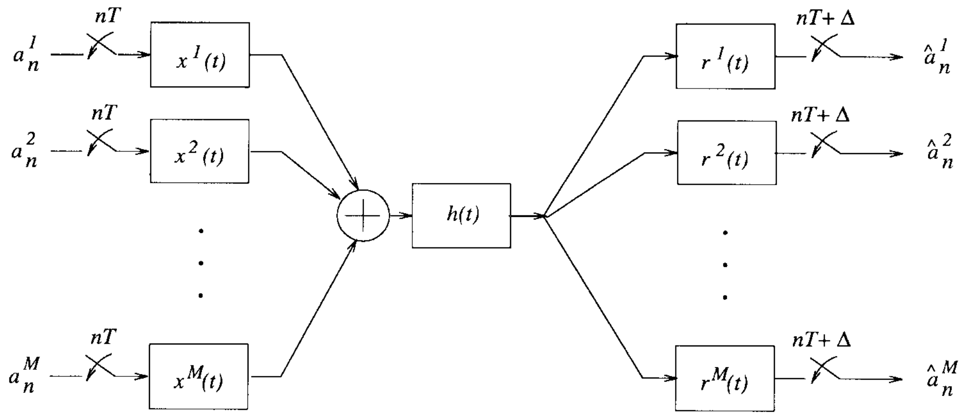

Fig. 5. Multiuser communication system.

There is an interesting duality between the consecutivity of the sample times and the contiguity[4] of the sample-rate folded channel support: 1) any M sampling times can be used provided that the sample-rate folded support is contiguous and at least M translates are available, since in this case $\tilde{\mathbf{W}}$ is full rank due to the determinant property of Vandermonde matrices and 2) any sample-rate folded support set having M translates available can be used provided that the M sampling times are consecutive, since in this case $\tilde{\mathbf{W}}$ is again full rank, for the same reason.

Finally, we remark that when $\dim \tilde{\mathbf{p}}_k = M$, the solution to (19) is unique. In addition, when there is only one nonzero term in each $\overline{P}_k^{(n)}$, then the equalizer set $r^k(t)$ is also unique. When $\dim \tilde{\mathbf{p}}_k > M$, then the system (19) is underdetermined and an infinite set of solutions exists, though among these there is a unique minimum-norm solution given by the pseudoinverse [14]. For the special case $\dim \tilde{\mathbf{p}}_k = N$, the minimum-norm solution to (17) is given by the right inverse

$$\mathbf{p}_k = \mathbf{W}^H(\mathbf{W}\mathbf{W}^H)^{-1}e^{-\hat{\imath}2\pi(k-1)fT_s}\mathbf{e}_k, \qquad 1 \le k \le M \tag{20}$$

but since $\mathbf{W}\mathbf{W}^H = N\mathbf{I}$

$$\mathbf{p}_k = \frac{1}{N}e^{-\hat{\imath}2\pi(k-1)fT_s}\begin{bmatrix} 1 \\ W^{(k-1)} \\ W^{2(k-1)} \\ \vdots \\ W^{(M-1)(k-1)} \end{bmatrix}. \tag{21}$$

V. Relation to MIMO Systems

The PCM modem problem described above has some interesting parallels to recent work in MIMO communication systems in which M users, each signaling at rate $1/T$ symbols/s, simultaneously communicate over a channel having bandwidth M/T Hz. Traditional techniques for the MIMO channel include code-division multiple access (CDMA), frequency-division multiple access (FDMA), and time-division multiple access (TDMA), but it is also feasible to use wide-band linear modulation, where each user employs the full available spectrum and equalizers are used to adaptively construct appropriately orthogonalized end-to-end impulse responses [15]–[18]. Here, we will show how time-staggering, which is a special case of linear modulation, can be used to achieve the same sort of orthogonalization and we will relate this to the problem described above.

Fig. 5 is a block diagram of an idealized multiuser data communication system in which M users transmit symbol sequences $\{a_n^1\}$, $\{a_n^2\}$, $\cdots$, $\{a_n^M\}$, each at rate $1/T$ symbols/s, using linear transmit filters $x^1(t)$, $x^2(t)$, $\cdots$, $x^M(t)$. Every user's symbols arrive simultaneously at the input to their respective transmit filters at times nT. The output of all of the transmit filters is summed and the resultant signal is transmitted over the linear time-invariant channel $h(t)$. Estimates $\{\hat{a}_n^1\}$, $\{\hat{a}_n^2\}$, $\cdots$, $\{\hat{a}_n^M\}$ of the original transmitted sequences are formed by passing the channel output signal through M parallel linear receive filters $r^1(t)$, $\cdots$, $r^M(t)$ and sampling the receiver output signals at times $nT + \Delta$, where Δ is the end-to-end system group delay.

Systems of this type have been treated in a variety of contexts [16]–[19] and it has been shown that if $h(t)$ has two-sided *contiguous* spectral support of at least M/T Hz, then it is possible to specify transmit and receive filters such that each of the M data streams can be recovered without interference from the others. More generally, a necessary and sufficient condition for elimination of interuser interference is given by the generalized zero-forcing (GZF) criterion [20]

$$\frac{1}{T}\sum_{l=-\infty}^{\infty} V_{km}\left(f + \frac{l}{T}\right) = \delta_{k-m}, \qquad 1 \le k,\, m \le M,\ |f| \le \frac{1}{2T} \tag{22}$$

where

$$V_{km}(f) = X_k(f)H(f)R_m(f) \tag{23}$$

is the frequency-domain transfer function of the kth transmitter, channel, and mth receiver in cascade. Since (22) must hold for any f, keep f fixed and let $L = L(f)$ denote the number of Nyquist translates $f_{l_i} = f + l_i/T$, $i = 1, \cdots, L$ for which $H(f_{l_i})$ does not vanish. Define $q_{l_i}^{(k)} \triangleq X_k(l_{f_i})H(f_{l_i})$ and the

[4] Contiguity here means that the support set is fully connected when depicted on the unit circle.

(row) vector

$$\mathbf{q}^{(k)} \triangleq [q_{l_1}^{(k)}, q_{l_2}^{(k)}, \cdots, q_{l_L}^{(k)}]. \tag{24}$$

Let

$$\mathbf{r}^{(m)} \triangleq \frac{1}{T}[R_m(f_{l_1}), R_m(f_{l_2}), \cdots, R_m(f_{l_L})]. \tag{25}$$

Then (22) states, for any $|f| \leq 1/2T$

$$\mathbf{q}^{(k)} \cdot \mathbf{r}^{(m)} = \delta_{k-m}, \qquad k, m = 1, 2, \cdots, M. \tag{26}$$

A necessary and sufficient condition for (26) to have a solution is that the M vectors $\mathbf{q}^{(k)}$ be linearly independent, which in turn imposes conditions jointly on $x^k(t)$, $r^k(t)$.

However, if the $x^k(t)$ or $r^k(t)$ are *a priori* restricted to be pure delays, then the multiuser system of Fig. 5 can be reinterpreted as a single-user system in which the user transmits just one symbol stream, with arbitrary time spacing between symbols. With this interpretation, the modem problem described above becomes a special case of the MIMO system.

For example, when the $x^k(t)$ are purely delay elements having distinct delays, then the multiple-branch transmitter in Fig. 6(a) can be reinterpreted as in Fig. 6(b), as a commutator sampling each of the transmitted symbol streams in turn, with delays τ_k between adjacent commutator positions. A further interpretation of this commutator is that of a single rate M/T transmitted symbol stream driving the channel, with fixed delays τ_k between successive symbols as in Fig. 6(c). If we choose $\tau_1 = \tau_2 = \cdots = \tau_k = 125\ \mu$s, then this interpretation is equivalent to the downstream direction of the PCM modem problem. In fact, the analysis in Section IV, establishing the support and sampling criteria for realizability of the $r^k(t)$, can be seen as a special case of the MIMO system GZF requirements (26) in which $x^k(t) \triangleq \delta(t-k\tau)$, $0 \leq k \leq M-1$, and $\tau = T/N$.

The MIMO interpretation also provides an intuitive justification as to why it is important that the nondata-bearing $N-M$ samples be forced to zero—if the unused samples were not zeroed, the transmit side of the downstream channel becomes equivalent to eight, rather than six, parallel 1-ksymbol/s data streams, and the nominal 3-kHz available bandwidth is insufficient to support these "additional" users without incurring interuser (or, in the single-user interpretation, intersymbol) interference.

Finally, we consider the case in which the transmitters (or receivers) are pure delays, but the τ_k are not uniform. This is not representative of the PCM modem problem, but it is useful and it follows easily from the preceding analysis. With arbitrary τ_k, the first user transmits at times $nT+\tau_1$ the second at $nT+\tau_2$, and so on, up to the Mth, who transmits at $nT+\tau_M$. Then, (13) becomes

$$\frac{1}{T}\sum_{n=-\infty}^{\infty} W_m^{-n} P_k\left(f+\frac{n}{T}\right) = \delta_{k-m} e^{-\hat{\imath}2\pi f\tau_k} \tag{27}$$

where $W_m = e^{-\hat{\imath}2\pi\tau_m/T}$. Defining $P_k^{(n)} = P_k(f+n/T)$, (15) becomes

$$\frac{1}{NT}\begin{bmatrix} \cdots & W_1 & 1 & W_1^{-1} & W_1^{-2} & \cdots & W_1^{-(N-1)} & \cdots \\ \cdots & W_2 & 1 & W_2^{-1} & W_2^{-2} & \cdots & W_2^{-(N-1)} & \cdots \\ \cdots & W_3 & 1 & W_3^{-1} & W_3^{-2} & \cdots & W_3^{-(N-1)} & \cdots \\ & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \\ \cdots & W_M & 1 & W_M^{-1} & W_M^{-2} & \cdots & W_M^{-(N-1)} & \cdots \end{bmatrix}$$
$$\cdot \begin{bmatrix} \vdots & \vdots & \vdots & & \vdots \\ P_1^{(-1)} & P_2^{(-1)} & P_3^{(-1)} & \cdots & P_M^{(-1)} \\ P_1^{(0)} & P_2^{(0)} & P_3^{(0)} & \cdots & P_M^{(0)} \\ P_1^{(1)} & P_2^{(1)} & P_3^{(1)} & \cdots & P_M^{(1)} \\ P_1^{(2)} & P_2^{(2)} & P_3^{(2)} & \cdots & P_M^{(2)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ P_1^{(N-1)} & P_2^{(N-1)} & P_3^{(N-1)} & \cdots & P_M^{(N-1)} \\ \vdots & \vdots & \vdots & & \vdots \end{bmatrix}$$
$$= \begin{bmatrix} e^{-\hat{\imath}2\pi f\tau_1} & 0 & 0 & \cdots & 0 \\ 0 & e^{-\hat{\imath}2\pi f\tau_2} & 0 & \cdots & 0 \\ 0 & 0 & e^{-\hat{\imath}2\pi f\tau_3} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & e^{-\hat{\imath}2\pi f\tau_M} \end{bmatrix}. \tag{28}$$

In this case, reduction to finite dimensional matrices is not possible in general, since the W_m do not necessarily possess the cyclic nature of W. On the other hand, any M columns of the matrix on the left-hand side are, in general, linearly independent. Similar to the earlier synchronous case, a unique solution exists if exactly M of the $H(f+k/T)$ are nonzero.

VI. Equalizer Adaptation Procedures

We next describe how appropriate equalizer settings for the PCM modem described above can be adaptively determined, assuming a sampling schedule and channel that meet the realizability criteria set forth in Section IV. With these criteria met, we know that a unique minimum-norm receiver exists which meets the GZF requirements, and we can then utilize standard minimum mean-squared error (MMSE) techniques to drive an appropriately defined error signal toward zero.

Recall that in the hybrid A/D arrangement of Fig. 1(b), user 2 is digitally connected to the PSTN. This feature facilitates adaptation since, in effect, user 2's transmitted digital data directly drives user 1's codec. This allows user 2 to force a known μ-law digital sequence into the codec, which can be used as a training sequence for adapting the downstream equalizers $r^k(t)$. Similarly, in the upstream direction, user 2 is capable of observing the digital representation of the sequence of μ-law encoded samples emerging from the codec, which can be used for adapting the $x^k(t)$. In what follows we also assume that for the purposes of initial handshaking and synchronization, user 1 and user 2 initially set up communication between themselves by utilizing conventional analog modem techniques over the available communication path and, in particular, that user 1 is capable of accurately determining the sampling rate at the codec.

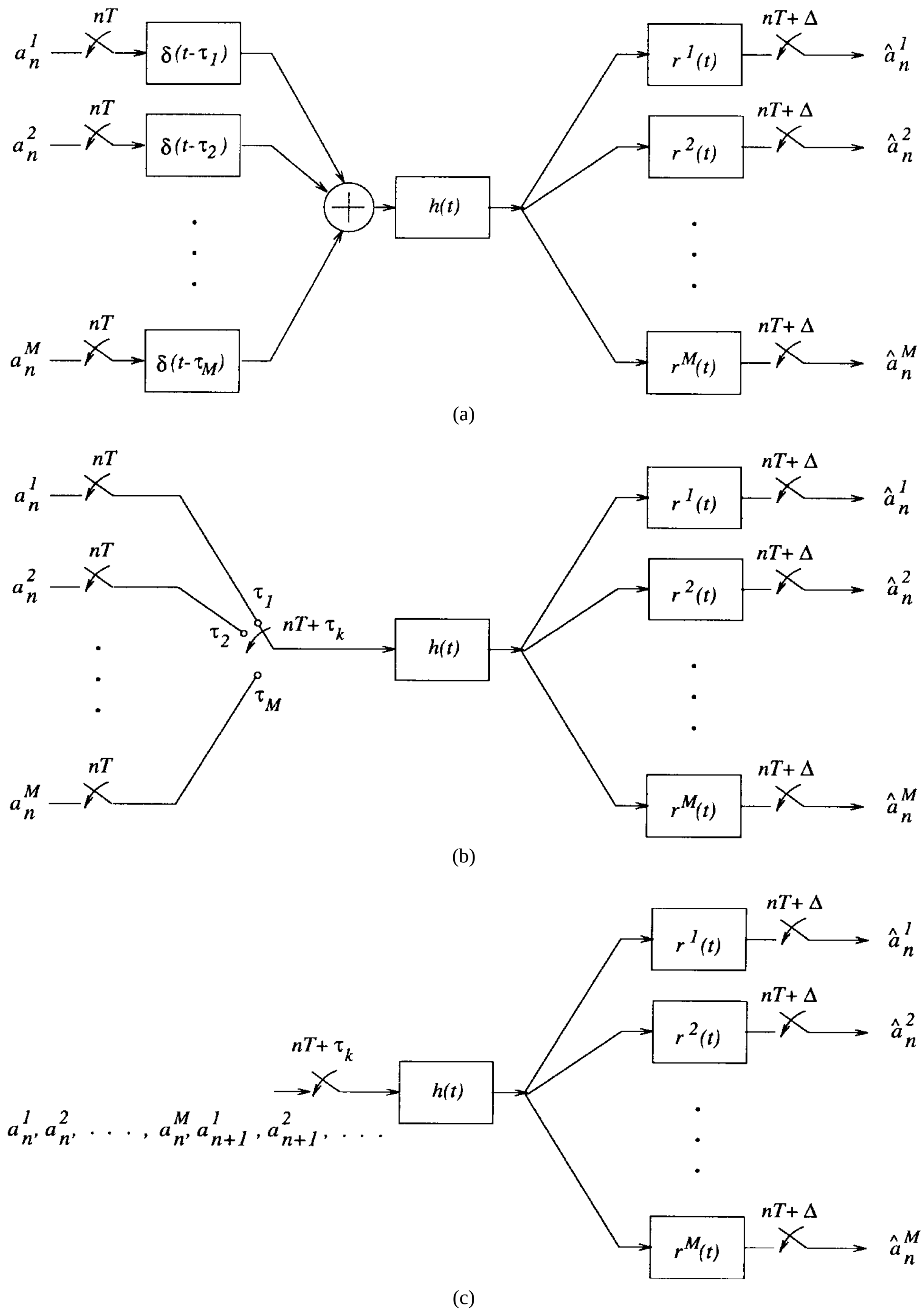

Fig. 6. Multiuser communication system as a set of receive-side equalizers.

We consider adaptation procedures for a general channel having unknown nonideal transmission characteristics. Continuing with the notation from Section IV, the codec samples uniformly at N/T Hz, and we assume for simplicity that the M data-bearing samples are consecutive. We further assume that the equalizers are implemented as finite-impulse response (FIR) filters with tap spacing T_s and that the realizability requirements of Section IV are satisfied for our choice of M, N, and f_s, so that we are assured of the existence of a set of equalizer filters which meet (15) or (26).

We first consider the equalization problem for the downstream direction shown in Fig. 3(a). Fig. 7(a) shows the equivalent discrete-time system with channel response $h(j)$ and the M discrete-time equalizers $r^k(j)$, $1 \leq k \leq M$. In this configuration $h(j)$ represents the cascade of the channel bank transmit filter, the analog loop impulse response, and user 1's modem front-end impulse response. As mentioned above, we assume that, during initial handshaking, user 1's modem has learned the codec sampling rate and that each of the constituent equalizer filters samples the received signal at this rate. The goal is to adaptively determine the $\{r^k(j), 1 \leq k \leq M\}$ such that the a_n^i are recovered at the output of $r^k(j)$.

This can be accomplished for the kth equalizer by arranging for user 2 to drive the codec with a known pseudorandom

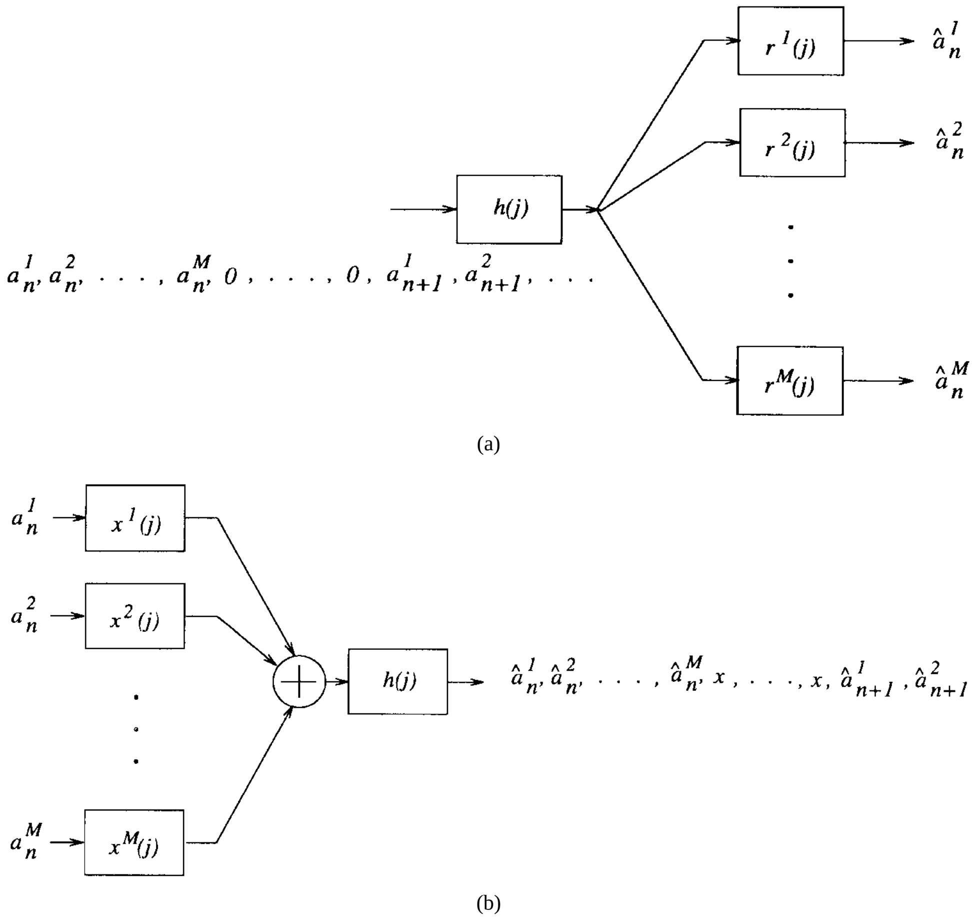

Fig. 7. Discrete-time equivalents of the PCM modem channel. (a) Downstream. (b) Upstream.

μ-law symbol sequence of the form

$$a_1^1\ a_1^2\ a_1^3\ \cdots\ a_1^M\ 0\ \cdots\ 0\ a_2^1\ a_2^2\ a_2^3\ \cdots\ a_2^M\ 0\ \cdots\ 0\ \cdots$$

where each N-sample block of the pseudorandom sequence consists of M nonzero samples a_n^i, $1 \le i \le M$, followed by $N - M$ zeros. The output y_n^k of the kth equalizer filter is computed at times nT (i.e., once every N input samples) and the coefficients are adapted based on the error signal $e_n^k \triangleq y_n^k - a_n^k$, using any of the well-known mean-squared error (MSE) minimization techniques, e.g., the least mean squares (LMS) algorithm. Using this approach, each of the $r^k(j)$ can be adapted independently of the others or, if desired, adaptation can be carried out for all $r^k(j)$ at once. The existence of a unique minimum-norm GZF solution guarantees that the $r^k(j)$ will converge toward this solution and that, in principle, any remaining error is due solely to implementation effects, e.g., number of available tap weights.

The corresponding equalizer adaptation problem for the upstream direction is shown in Fig. 7(b). The adaptation procedure here is somewhat different than that for the downstream case, primarily because in the upstream direction the channel follows the equalizer, as shown in Fig. 8(a). As a consequence, the outputs from the equalizer–channel cascade which are necessary for computing the adaptation error signal e_n^k are not directly available to user 1 where they are required to update the $x^k(j)$. Furthermore, even if these outputs were, say, returned instantaneously to user 1 from the codec end, there would still be the case that adaptive updates made to the $x^k(j)$ would affect e_n a relatively long time after they are made, due to the group delay of $h(j)$. This delay has undesirable consequences for many adaptation techniques, e.g., reduced stability, requiring small stepsizes, and, consequently, slow convergence.

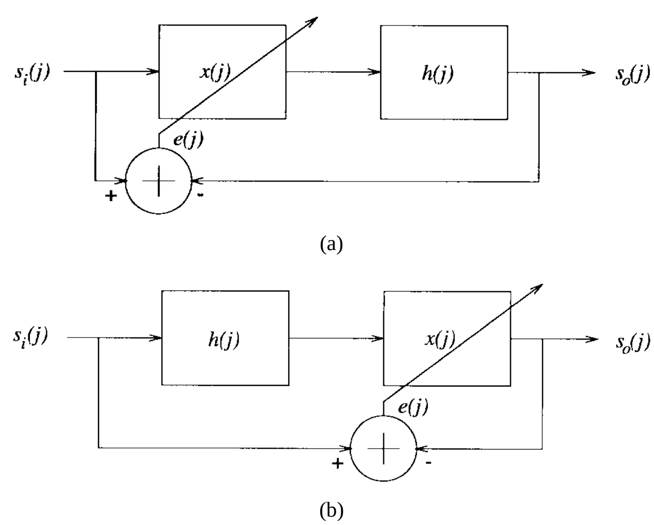

Fig. 8. Equivalent configurations for transmitter equalization, but not for adaptation.

An alternative approach is to arrange for the transmit equalizers to be initially synthesized by user 2 during a training period and then transferred to user 1, where they would be used for normal data transmission, as shown in Fig. 8(b). Linearity ensures that this rearrangement is transparent.

The transmit equalizers can be adapted in this way by having user 1 drive the channel in Fig. 7(a) directly with an input consisting of the pseudorandom sequence

$$a_1^1\, a_1^2\, a_1^3\, \cdots\, a_1^M\, 0\, \cdots\, 0\, a_2^1\, a_2^2\, a_2^3\, \cdots\, a_2^M\, 0\, \cdots\, 0\, \cdots.$$

Then, at user 2's modem, the relocated transmitter filters can be designed based on the above known sequence. Here, however, in contrast to the adaptation of the receivers, we must adapt the coefficients of the $x^k(j)$ on a *sample* basis, rather than every N samples, because here we need to force the composite impulse response $x^k(j) * h(j)$ to zero at all sampling times which are used by other transmitters. For example, transmitter filter $x^1(j)$ would be adapted based on the difference between its T_s-sampled output and the desired sequence

$$a_1^1\, 0\, 0\, \cdots\, 0\, x\, \cdots\, x\, a_2^1\, 0\, 0\, \cdots\, 0\, x\, \cdots\, x\, \cdots$$

where x denotes the $N - M$ nondata-bearing samples; the coefficients of the $x^1(k)$ are updated based on the error signal computed from all samples except these. Similarly, the desired response for $x^2(k)$ is

$$0\, a_1^2\, 0\, \cdots\, 0\, x\, \cdots\, x\, 0\, a_2^2\, 0\, \cdots\, 0\, x\, \cdots\, x\, \cdots$$

and for $x^3(k)$

$$0\, 0\, a_1^3\, \cdots\, 0\, x\, \cdots\, x\, 0\, 0\, a_2^3\, \cdots\, 0\, x\, \cdots\, x\, \cdots$$

and so on. Finally, once the transmit equalizers are designed in this way by user 2, the postchannel equalizer configuration [Fig. 8(b)] is transformed into the prechannel equalizer configuration [Fig. 8(a)] by transferring the FIR equalizer coefficients to user 1.

We note that, since the transmit preequalizer is compensating for the loss in the loop to arrive at the network codec at a constant reference signal level, loops with high loss will yield high transmitted signal levels at the modem transmitter output. In practice the transmitted signal power is limited by regulatory constraints in most countries. Therefore, in the presence of a loop with very high loss, the modem will have to limit the constellation it uses, hence the data rate, to remain within power constraints. This limitation is quite acceptable for typical loops which exhibit 0–10 dB of signal attenuation.

VII. Concluding Remarks

In this paper we have modeled the channel as strictly bandlimited. Most codecs in the field allow significant energy across the whole band of 0–4 kHz. There is always an exact null at DC due to transformer coupling, and attenuation at 4 kHz ranges from about 10 to tens of decibels. In addition, the upstream codec filters incorporate a notch at 60 Hz to suppress power line interference. To approach channel capacity on many of these channels using the multiple equalizer technique presented here, one may need to employ large values for the integers M and N of Section IV. Practical implementation of a PCM modem with limited available memory and processing power may limit the feasible values of M and N to values somewhat lower than what would be desired to approach channel capacity. In such cases with generous bandwidth available, other equalization schemes such as partial response [21] can yield higher achievable rates in a practical implementation.

In addition, we wish to point out that, due to its complexity, we did not consider the approach of faster-than-Nyquist signaling (see, e.g., [22]). We also wish to point out that there are alternative rate formulations in the literature for related problems (see, e.g., [23]).

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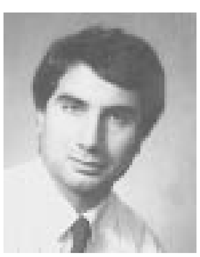
Ender Ayanoglu (S'82–M'85–SM'90–F'98) received the B.S. degree from Middle East Technical University, Ankara, Turkey, in 1980, and the M.S. and Ph.D. degrees from Stanford University, Stanford, CA, in 1982 and 1986, respectively, all in electrical engineering.

Since 1986 he has been with Bell Laboratories, Holmdel, NJ. His current research interests are in broadband integrated communications, wireless ATM, multiwavelength optical networking, and general source and channel coding applications in communication networks.

Dr. Ayanoglu is the Vice Chair of the IEEE Communication Society Communication Theory Technical Committee and the Editor for Communication Theory and Coding Applications of the IEEE TRANSACTIONS ON COMMUNICATIONS. He received the IEEE Communications Society Stephen O. Rice Prize Paper Award in 1995 and the IEEE Communications Society Best Tutorial Paper Award in 1997.



Nuri R. Dagdeviren was born in Konya, Turkey, in 1961. He received the B.S. degree from Bogazici University, Istanbul, Turkey, in 1982, and the Ph.D. degree from Massachusetts Institute of Technology, Cambridge, in 1985, both in physics.

He was a Research Fellow in theoretical physics at the California Institute of Technology from 1985 to 1987. Since 1987 he has been with Bell Laboratories, Holmdel, NJ. He has worked in the fields of operations support systems, ISDN networking and signaling protocols, and, since 1991, modems. His current research interests are in digital signal processing for communications applications.

Glenn D. Golden (M'89) received the B.S. degree in physics from Stevens Institute of Technology, Hoboken, NJ, in 1976.

He has worked in the fields of data communications and signal processing, and has been involved in both research and development in a wide variety of areas, including voice recognition and speaker verification, high-speed modems, telephone switching and transmission systems, data compression, neural networks, and adaptive antenna arrays. He has worked with more than a dozen firms throughout the U.S. including ITT, Lockheed, RCA, AT&T, Lucent Technologies, and numerous smaller companies, and he holds patents in a variety of areas including equalization, echo cancellation, modem design, adaptive array processing, and other wireless techniques. He is currently with the Wireless Communications Research Department, Bell Laboratories, Holmdel, NJ, as a Member of Technical Staff.

James E. Mazo (M'65–SM'90–F'91) received the B.S. degree in physics from the Massachusetts Institute of Technology, Cambridge, in 1958, and the M.S. and Ph.D. degrees in physics from Syracuse University, Syracuse, NY, in 1960 and 1963, respectively.

Upon completion of his studies, he joined the Department of Physics, University of Indiana, Bloomington, as a Research Associate. In 1964 he joined Bell Telephone Laboratories, Inc., Holmdel, NJ, working on problems of data transmission. In 1972 he transferred to the Mathematical Sciences Research Center, Bell Laboratories, Murray Hill, NJ, where he continued research on all aspects of communication theory. He spent 1985 supervising the Data Theory group at AT&T Information Systems, returning to his position at AT&T Bell Laboratories in 1986. In July 1993 he became Head of the Mathematics of Communications Research Department at Bell Laboratories, Murray Hill, NJ. During his entire career, he has been most attracted to the more theoretical or mathematical aspects of communication engineering. He has more than 40 published papers. He has contributed successfully to solving problems in nonlinear noise theory, FM transmission, equalization theory, digital filtering, trellis coding, and modem design, being co-inventor of the K56Flex modem.

Dr. Mazo was the co-recipient of the 1996 S. O. Rice Best Paper Award.