Effects of Wavelength Routing and Selection Algorithms on Wavelength Conversion Gain in WDM Optical Networks

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***Abstract*— Wavelength-division multiplexing (WDM) technology is emerging as the transmission and switching mechanism for future optical mesh networks. In these networks it is desired that a wavelength can be routed without electrical conversions. Two technologies are possible for this purpose: wavelength-selective cross-connects (WSXC) and wavelength interchanging cross-connects (WIXC), which involve wavelength conversion. It is believed that wavelength converters may improve the blocking performance, but there is a mix of results in the literature on the amount of this performance enhancement. In this paper we use two metrics to quantify the wavelength conversion gain: the reduction in blocking probability and the increase in maximum utilization, compared to a network without converters. We study the effects of wavelength routing and selection algorithms on these measures for mesh networks. We use the overflow model to analyze the blocking probability for wavelength-selective (WS) mesh networks using the first-fit wavelength assignment algorithm. We propose a dynamic routing and wavelength selection algorithm, the least-loaded routing (LLR) algorithm, which jointly selects the least-loaded route–wavelength pair. In networks both with and without wavelength converters the LLR algorithm achieves much better blocking performance compared to the fixed shortest path routing algorithm. LLR produces larger wavelength conversion gains; however, these large gains are not realized in sufficiently wide utilization regions and are diminished with the increased number of fibers.**

***Index Terms*— Optical networks, routing, wavelength conversion, wavelength selection.**

I. Introduction

IN RECENT YEARS, there has been significant research in studying all-optical networks which provide optical transmission and switching. Wavelength-division multiplexing (WDM) and optical switching provide networks with increased transmission bandwidth and flexibility. This flexibility, provided by the transparency to signal format and bit rate, may enable cost-effective high-capacity switching as well as simplified network management. This approach will work with existing optical transmission and electrical switching equipment in a multilayer transport network architecture, and could be appropriate where the traffic volume between nodes is high.

Recent research has shown that the combination of WDM transmission, optical multiplexing/demultiplexing, and optical space switching may be used to implement all-optical cross-connect networks. These networks can be more efficient and cost-effective than their electrical counterparts due to advantages in building network elements of large size and capacity [1].

Since a realistic optical packet-switching technology is not available today, research in all-optical networks is mostly confined to circuit switching, for which each connection is assigned a route in the network and a wavelength on each link along the route. In this paper we consider two types of all-optical circuit-switched networks: *wavelength-selective* (WS) and *wavelength-interchangeable* (WI) networks. In a WS network a connection can only be established if the same wavelength is available on all links between the origin and the destination nodes. This means that a connection request can be blocked even if there are available wavelengths on all links. The blocking probability can be reduced by allowing the connection to change from one wavelength to another at an intermediate cross connect, which is known as *wavelength conversion*. In this paper a network in which all cross connects have wavelength conversion capability (from any wavelength to any other wavelength) is called a WI network. We quantify the benefits of wavelength conversion for mesh networks with different routing and wavelength selection algorithms. Two metrics are used in this paper to measure the wavelength conversion gain: the *blocking probability gain* corresponds to the reduction in blocking probability, and the *utilization gain* measures the increase in network utilization.

Wavelength-routed optical networks have been under investigation in a number of research projects in the U.S. [2]–[5], Europe [6]–[8], and Japan [9]. Analysis of the performance improvement with wavelength converters is very important for the design of optical networks. In general, the performance benefits of wavelength conversion depends on many factors such as topology (in particular, network size and connectivity), number of wavelengths per fiber, number of fibers per link, traffic load, routing, and wavelength selection algorithms. Fully connected networks constitute one extreme of topology

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where there is no gain with the shortest path routing algorithm. At the other extreme is the ring topology for which the gain is also relatively small [10]–[12]. It has been shown that intermediately connected networks, such as a mesh, have the largest gain [13]. This observation is consistent with the two examples in the literature with extremely high wavelength conversion gains (the ratio of blocking probabilities with and without wavelength conversion as high as 10^9) obtained using the mesh–torus topology [10], [12].

The effect of network topology and number of wavelengths on the wavelength conversion gain has been studied for single-fiber networks using shortest path routing and random wavelength selection algorithms in [10]–[12], [14], and [15]. The analysis in [14] is extended to multifiber networks [11], [13]. These models predict that the gain drops exponentially with the number of fibers. It will be shown by simulations in Section IV that the exponential model accurately predicts the conversion gain for a moderate number of fibers.

Routing and wavelength selection algorithms that have been studied for optical networks can be classified as follows. The *fixed shortest path routing* algorithm uses a predetermined shortest path each time a connection is established [16]. With the *alternate routing* algorithm, a set of alternate routes is considered for availability sequentially in a fixed order [16]. The *random wavelength selection* algorithm [10], [14] selects a wavelength randomly among currently available ones. With the *first-fit wavelength selection* algorithm [17], [18], the available wavelength with the smallest index is chosen, whereas the *most-used wavelength selection* algorithm [18], [19] selects the available wavelength which is currently utilized on the largest number of fibers. The random rule distributes the traffic randomly so that average wavelength utilizations are balanced. The first-fit rule tries to pack wavelengths according to a fixed order, whereas the most-used rule packs wavelengths according to their utilizations.

In the literature the routing and wavelength selection problems are decoupled in order to simplify both problems at the expense of performance, i.e., the routing problem is solved independent of the information on which wavelengths are utilized on the path, and vice versa. In this paper, we present the *least-loaded routing* (LLR) algorithm, which is a dynamic routing algorithm that jointly selects the route–wavelength pair for each connection. This selection is made by using the current state of the network so that further congestion in the already heavily loaded parts of the network can be avoided. The adaptivity of the algorithm to the network state combined with the joint nature of the route and wavelength selection process provide enhanced performance.

In [20], blocking probabilities for a 24-node WS mesh network are reported, where wavelength assignment and path selection are performed by using separate heuristic algorithms. The benefits of wavelength conversion are studied in [21] for randomly generated single-fiber networks with 16–1000 nodes using the fixed shortest path routing and first-fit wavelength selection algorithms. A wavelength conversion gain corresponding to 10%–40% increase in wavelength reuse (utilization) is shown. However, these gains are obtained at a blocking probability of 10^{-2}, which is very high. In [22] a six-node network is used with fixed and dynamic alternate routing (using a small set of alternate paths for each origin–destination (o–d) pair) and the random and first-fit wavelength selection algorithms. Blocking probabilities with and without wavelength converters are obtained by simulations. The routing strategy used in [22] is not adaptive to network load, and the simulations are performed only at a particular value of the offered load.

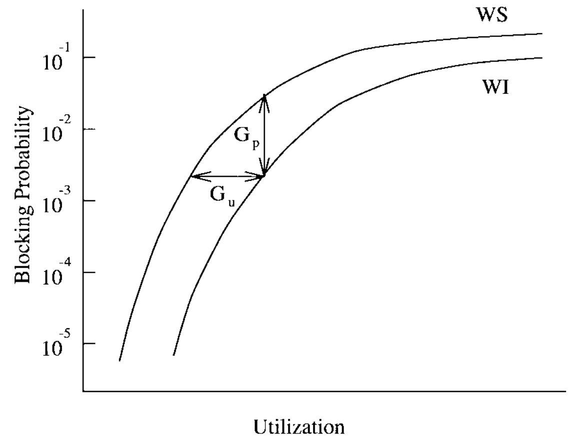

Fig. 1. The description of the utilization gain G_u and the blocking gain G_p.

Adaptive routing and wavelength selection with unconstrained path sets have been considered in [18], where all wavelengths are searched sequentially until an available path is found over one wavelength. This routing and wavelength assignment algorithm is not sensitive to the network utilization level, i.e., it selects a path independent of the distribution of its available resources as long as the path is available. However, routing of a connection over a path that is already congested leads to further congestion of links on that path, which then can lead to the blocking of future connection requests. A good adaptive routing algorithm should consider the utilization level on the links to avoid additional load on congested parts of the network.

In this paper we quantify the benefits of wavelength conversion for different routing and wavelength selection algorithms by means of analysis and simulations. We use two metrics to measure the merits of wavelength conversion. The utilization gain G_u is the ratio of maximum offered loads for WI and WS for achieving a given blocking probability [14]. Similarly, we define the blocking probability gain G_p as the ratio of blocking probabilities for WS and WI networks for a given traffic load (see Fig. 1). The routing and wavelength selection algorithms used in optical networks are very critical in determining G_u and G_p. As shown through the examples in Sections II, III, and IV, for the same network topology and traffic load, G_p may be significantly different from one algorithm to another.

In the literature there are extensive comparisons of analytical and simulation-based methods which are used to study the performance of different routing and wavelength selection algorithms for optical networks. Simulation studies in the literature show that the blocking performance of a WS network is improved considerably when wavelength packing type algorithms such as the first-fit rule are employed for wavelength assignment [11], [18], [22]. On the other hand, most analytical studies for obtaining the blocking probability P_b for WS networks assume that all wavelengths have identical traffic loads, which is the case for only some wavelength selection algorithms, e.g., the random selection rule [10], [12], [14], [15]. With the first-fit rule, the traffic load on each wavelength decreases as the wavelength number increases. An analytical model for computing the blocking probability for WS networks with the first-fit algorithm was presented recently in [18]. However, this model does not consider the peakedness of the blocked traffic on individual wavelengths.

We here apply the equivalent random method which is used in the literature to analyze the blocking probability for circuit-switched networks with non-Poisson offered traffic [16], [24]. This analytical model for computing P_b with the first-fit algorithm is called the *overflow model*. In the overflow model it is assumed that all traffic is offered to the first wavelength and the overflow traffic from wavelength k is offered to wavelength $k+1$. The traffic overflowing from the last wavelength is the blocked traffic. The second moment of the overflow traffic is used in computing the blocking probability for each wavelength. The numerical studies presented in this paper using the overflow model show a close match between the analytical and simulation results.

The model presented in [14] and developed later in [11] and [15] provides equations which can be used to study the qualitative behavior of the utilization gain G_u^p for path p as a function of the path length, number of wavelengths, and number of fibers when the random wavelength selection algorithm is used. The utilization gain is upper bounded as

$$G_u^p \leq \left(\frac{H_p}{L_p}\right)^{1/M} \quad (1)$$

where H_p is the number of links on path p, L_p is the average number of links shared by paths intersecting with p (*interference length*), and M is the number of fibers per link [11]. An approximation for G_u^p as $P_b \to 0$ is obtained as

$$\lim_{P_b \to 0} G_u^p = \left(\frac{H_p^{1-(1/K)}}{L_p}\right)^{1/M} \quad (2)$$

where K is the number of wavelengths per fiber [13].

The effect of the topology and the routing algorithm on G_u can be studied using (1) and (2). As the network gets larger, i.e., larger *average path length* H, G_u increases. More importantly, shortest path routing which minimizes H for a given network reduces G_u. We observe from (2) that as the average interference length L gets larger, G_u decreases. The interference length not only depends on topology but also is determined by the routing algorithm. Shortest path algorithms such as Dijkstra or Bellman–Ford [23] produce paths sharing multiple links (large L).

On the other hand, routing algorithms which use a larger number of paths per node pair (such as k shortest paths) produce smaller L, as shown in Fig. 2. The values of H and L are obtained for the 30-node mesh network in Fig. 3 with a uniformly distributed traffic, i.e., all node pairs have the same amount of traffic. The values of H and L in this figure are computed using the following procedure:

1) for each k, the path set comprising the k shortest paths is obtained;
2) H is computed by taking the average over all paths in the path set;
3) L is obtained by correlating each path with all paths in the path set, and then by taking the average over all paths.

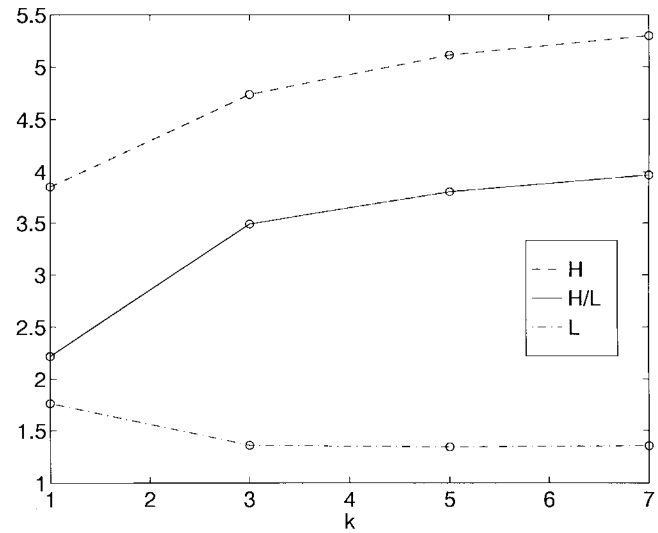

Fig. 2. H, L, and H/L versus k.

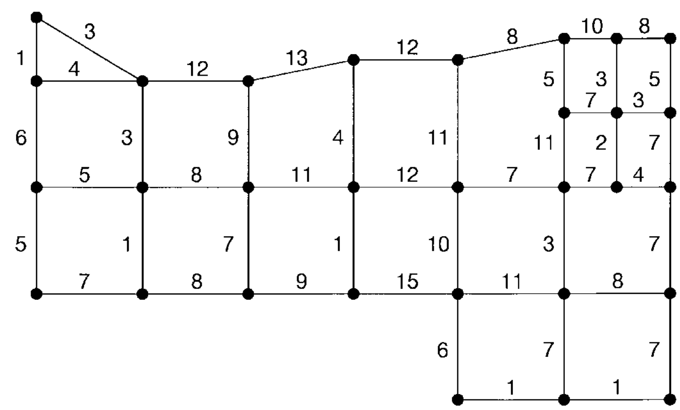

Fig. 3. The 30-node mesh network used in the simulations.

Finally, the effect of K, the number of wavelengths, is weak as $P_b \to 0$, much weaker than M, which reduces G_u exponentially. The strong dependence on M is significant since with the current WDM technologies there are technological and economical advantages of having multiple fibers on each optical link.

In our simulations we consider two general classes of routing algorithms: fixed and dynamic. The shortest path (having the minimum number of links) between the origin and destination nodes is used to establish a connection request in the fixed shortest path routing. If the connection cannot be established along the shortest path, the connection request is blocked. With the fixed routing strategy for the WS network, the random, most-used, and first-fit wavelength selection algorithms are used for single-fiber networks. For the multifiber network, the utilization level on each wavelength can be used to improve the performance. In Section II two wavelength selection algorithms that select the most lightly loaded wavelength along the shortest path for the multifiber case are proposed.

With the dynamic routing strategy, we choose the path and the wavelength jointly among the set of all available wavelengths along k shortest paths between the origin and destination nodes. We propose an algorithm which selects the least-loaded path–wavelength pair among alternatives, called the LLR algorithm, which will be discussed in Section IV. Our simulation results show that the blocking probabilities for the WS and WI networks can be reduced considerably with the LLR algorithm while obtaining larger conversion gains compared with the fixed shortest path routing algorithm. However, these gains reduce rapidly with increasing the load and the number of fibers.

The paper is organized as follows. The simulation results obtained by using the fixed shortest path routing with different wavelength selection algorithms are presented in Section II. In Section III an analytical model to obtain the blocking probability in WS mesh networks using the first-fit wavelength assignment algorithm is introduced. These analytical results are also compared with simulations for the first-fit and random wavelength assignment algorithms. The adaptive routing algorithm LLR is presented in Section IV, and the simulation results are discussed. Section V concludes the paper.

II. Shortest Path Routing

In the fixed shortest path routing algorithm, the set of shortest paths between all node pairs is computed in advance and stored in routing tables at each node. When a connection request arrives, the shortest path between the o–d pair is used to make the connection. If the connection cannot be established along this path, it is blocked and cleared. If the connection can be established using the shortest path for the WS network, a wavelength is selected among the set of available wavelengths. The first-fit, most-used, or random wavelength selection algorithms are studied in this section for single-fiber networks.

When the network has multiple fiber links, the usage level at each wavelength can be used to determine the link load. For the multifiber case, we use algorithms that choose the wavelength based on the load values along the shortest path. Two such algorithms are proposed in this section.

Let M_l denote the number of fibers on link l and let A_{lj} denote the number of fibers (or optical connections) for which wavelength j is utilized on link l. The set of available wavelengths along the shortest path p is denoted by S_p. We use the following two dynamic wavelength selection algorithms for the multiple-fiber case.

- *Least-loaded (LL)*: The minimum index wavelength j in S_p that achieves

$$\max_{j\in S_p} \min_{l\in p} [M_l - A_{lj}]$$

  is selected.

- *Minimum sum (MS)*: The minimum index wavelength j in S_p that achieves

$$\min_{j\in S_p} \sum_{l\in p} \frac{A_{lj}}{M_l}$$

  is selected.

The LL rule selects the wavelength that has the largest residual capacity on the most loaded link along p. The MS algorithm chooses the wavelength that has the minimum average utilization. Both MS and LL rules select the most used wavelength when multiple wavelengths are tied, hence they reduce to the most-used rule in the single-fiber case. The performance of shortest path routing algorithms have been investigated in the literature for random [10]–[12], [14], [15], first-fit [13], [21], [22], and most-used [18], [19] wavelength selection rules. The LL and MS wavelength selection rules for the multifiber case are proposed in this paper.

For the simulations in Sections II and IV, we use the 30-node mesh network given in Fig. 3. The geographical locations of these nodes are selected to reflect the locations of major U.S. cities. The connection requests arrive at each node according to a Poisson process with rate λ and with a destination selected randomly and uniformly among other nodes. We assume throughout this paper that all connections are full duplex and that each fiber supports full duplex lightpaths. Once a connection is established, it holds a wavelength on each link along the shortest path for an exponentially distributed time with unit mean.

The network used in the simulations has either a single fiber for each link, or has multiple fibers. Each fiber has $K = 8$ wavelengths, which is selected in accordance with the Multiwavelength Optical Networking (MONET) [5] architecture. In the multifiber network, the network design is carried out by using a traffic demand matrix $T = [t_{ij}]$, where t_{ij} is the number of wavelength demands between nodes i and j. This demand is uniformly random with mean m, i.e., t_{ij} takes equally probable integer values in $[0, 2m]$. The traffic demand matrix T does not represent the actual traffic, but instead it has to be obtained from the forecasts of the traffic which will be carried by the designed network.

The number of fibers for each link is engineered by routing each traffic demand along the shortest path p. In the design of the WS network the wavelength for each connection is assigned starting from the longer paths in order to reduce wavelength conflicts [17]. For each connection, the wavelength j that minimizes $\sum_{l\in p} A_{lj}$ is selected, i.e., the least-used wavelength along the shortest path is chosen. Once wavelengths are assigned to all connections, the number of fibers for each link is given by

$$M_l = \max_j A_{lj}.$$

After the link sizes are engineered, connections are established and torn down dynamically and connection blocking statistics are gathered after the system reaches the steady state. In our simulations for the multiple-fiber case we use the 30-node mesh network designed using WS cross-connects (WSXC) at each node as described above. Performance of the

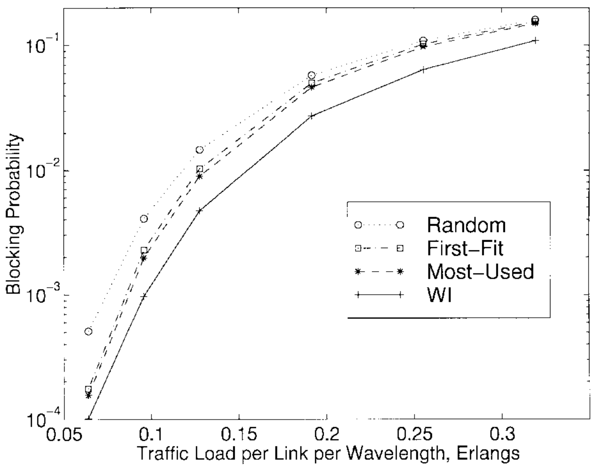

Fig. 4. P_b versus traffic load in Erlangs for the single-fiber network with the random, first-fit, and most-used wavelength assignment algorithms.

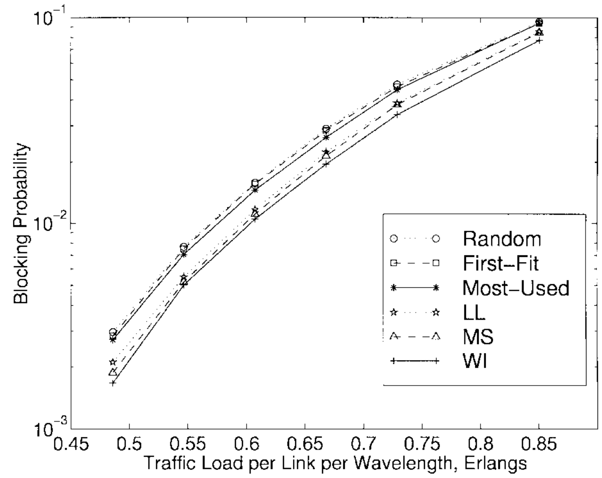

Fig. 5. P_b versus traffic load in Erlangs for the multiple-fiber network ($m = 0.5$) with the MS, LL, most-used, first-fit, and random wavelength selection algorithms.

five wavelength assignment algorithms for the WS network is compared to the performance of the WI network which has the same number of fibers per link as the WS network.

The blocking probability is plotted in Fig. 4 for the single-fiber case as a function of the traffic load. The load is expressed by the link utilization per wavelength given by

$$\rho = \frac{N\lambda H}{JMK} \tag{3}$$

where N is the number of nodes, H is the average number of links per path, J is the number of links, and M is the average number of fibers per link. We observe that the first-fit algorithm performs much better than the random algorithm at low loads, whereas the difference between the two algorithms is marginal at higher utilizations. The most-used algorithm slightly outperforms the first-fit algorithm. The utilization gains are $G_u = 1.13$ with the most-used algorithm and $G_u = 1.31$ with the random algorithm at $P_b = 10^{-3}$. Since most of the call blockings at lower utilizations are caused by wavelength conflicts for the WS network, the selection algorithm plays an important role in the low blocking probability region. As the network load increases, most of the call blockings are caused by insufficient bandwidth whether there is wavelength conversion or not. Consequently, G_p gets smaller.

The blocking probability is plotted in Fig. 5 for the multiple-fiber network with different wavelength selection algorithms. The network is designed for $m = 0.5$, i.e., $t_{ij} = 0$ or 1 with equal probability, and the ensuing network has an average of $M = 5.25$ fibers per link. Both adaptive wavelength selection algorithms perform much better than the random, first-fit and most-used selection rules. The order of performance between the random, first-fit, and most-used algorithms is the same as the single-fiber case; however, the performance differences between these algorithms are much smaller. Among the two adaptive wavelength selection rules, the MS is slightly better, especially at lower utilizations. The utilization gains are $G_u = 1.06$ with the most-used algorithm and $G_u = 1.01$ with the MS algorithm at $P_b = 3 \times 10^{-3}$. The wavelength conversion gain for the multifiber case is significantly less than the single-fiber case, as predicted by [11] and [13]. We study the behavior of the utilization gain as a function of M in Section IV.

In our simulations we came up with scenarios where WS is less blocking than WI, especially when the network is heavily loaded. Similar observations were also presented in the literature, e.g., [10] and [19]. This is primarily due to the fact that WI networks can accommodate more connections with long paths since wavelength conversion avoids conflicts, which is more of a problem with long paths. By accepting a long path when the network is congested, WI causes rejection of several subsequent connection requests requiring shorter paths. In WS optical networks the wavelength constraint acts as a protection mechanism that blocks more connection requests with long paths, especially when the network is heavily loaded. Consequently, the overall blocking probability is reduced and WS networks may perform better than WI networks under heavy load when no admission control strategy is employed. The problem of admission control for WS networks in order to achieve low blocking probability while sustaining a high level of fairness has not been studied yet. The blocking performances of WS and WI networks need to be compared when admission control algorithms that are separately optimized for both networks are used.

Among the different rules we discussed above, the wavelength-packing type algorithms such as the first-fit rule perform better than the random rule as shown by the simulation results given in Figs. 4 and 5, especially when the number of fibers per link is small. The performance of the first-fit rule is mostly evaluated in the literature by simulation techniques [11], [22], and there are recent analytical models to evaluate the blocking probability for WS networks with alternate routing and first-fit wavelength selection [18]. However, this model does not consider the peakedness of the overflow traffic, and instead uses the Erlang-B formula. In the next section we present an analytical model to compute the blocking probability for WS networks employing the first-fit algorithm where the blocking probability is computed by using both the mean and the variance of the overflow traffic from each wavelength.

III. Overflow Model for the First-Fit Wavelength Assignment Algorithm in WS Mesh Networks

Without wavelength interchangers a connection must use the same wavelength on every link of the path. As shown by the simulations in Section II, the wavelength assignment algorithm plays an important role in the performance of these networks. In this section we develop a model for analyzing the blocking probability in a WS network for the first-fit wavelength assignment algorithm. We present the overflow model and evaluate its performance. The blocking probability obtained using the overflow model is close to the blocking probability obtained from simulations of the first-fit wavelength assignment algorithm for a 16-node mesh–torus network.

The majority of the previous work on the analysis of blocking probability for the WS networks assumes that the traffic streams offered to individual wavelengths on a fiber are independent and identically distributed [10], [13], [14]. This is the case when wavelengths are assigned randomly among available wavelengths, i.e., the random wavelength selection rule. Neither the assumption of independence nor the assumption of identical distribution is valid for the first-fit rule, where the utilization on individual wavelengths on a link decreases with the wavelength number.

The analysis of the first-fit algorithm is complicated by this uneven load on wavelengths. New techniques are necessary to analyze the blocking probability for WS mesh networks using the first-fit rule. An analytical tool to model this wavelength assignment algorithm is the overflow model which is presented in this section. We assume that the traffic offered to wavelength 1 on a link is equal to the total traffic offered to the link. The traffic which cannot be carried on wavelength 1 (overflow traffic from wavelength 1) is offered to wavelength 2. In general, the overflow traffic from wavelength k is offered to wavelength $k+1$, and the overflow traffic from wavelength K is the blocked traffic from the link, where K is the number of wavelengths. One of the difficulties in the analysis of the first-fit rule is the bursty nature of the overflow traffic from each wavelength, which prevents the direct application of the Erlang-B formula. The overflow model uses both the first and second moments of the overflow traffic to compute the blocking probability.

A. Analysis of Blocking Probability by Using the Overflow Model

Traffic offered to wavelength 1 for any path p is equal to the total traffic offered to path p. We assume that connection requests arrive to a node according to a Poisson process with rate λ with uniformly selected destinations and exponentially distributed holding times with mean $1/\mu$. We also assume that a single path is used for each source–destination pair. The traffic A_1^p offered to wavelength 1 for any path p is given by

$$A_1^p = \frac{\lambda}{\mu(N-1)} \tag{4}$$

where N is the number of nodes in the network.

Although the offered traffic for wavelength 1 is Poisson, the overflow traffic from each wavelength is bursty. Therefore, the assumption that the traffic offered to each link for any wavelength is Poisson underestimates the link blocking probability. Instead, we apply the equivalent random method [16], [24] which uses both the mean A_{lk} and the variance V_{lk} of the bursty traffic offered to link l for wavelength k, $k \geq 2$ to obtain the link blocking probability B_{lk}. Moment-matching techniques, such as the equivalent random method, have been used to analyze blocking probabilities in telephone networks with alternate routing (see [25] and [26] for a review).

The variance V of the overflow traffic from a system of M channels with Poisson-offered traffic is given by the Brockmeyer model [16]

$$V = \mathcal{O}\left(1 - \mathcal{O} + \frac{A}{M + 1 - A + \mathcal{O}}\right) \tag{5}$$

where A is the mean offered traffic and $\mathcal{O}$ is the mean overflow traffic [16]. Mean overflow traffic is given by $\mathcal{O} = AE(A, M)$, where $E(A, M)$ is the Erlang-B formula. The number of channels M in (5) corresponds to the number of fibers per link since the number of channels for each wavelength on a link is given by the number of fibers on that link.

In the overflow model the offered traffic for wavelength $k+1$ on path p is given by the overflow from wavelength k on p

$$A_{k+1}^p = \mathcal{O}_k^p = A_k^p B_k^p \tag{6}$$

where B_k^p denotes the blocking probability on path p for wavelength k. We assume that for a given path p and wavelength k the events corresponding to blocking of wavelength k on each link along p are all independent, i.e.,

$$B_k^p = 1 - \prod_{l \in p} (1 - B_{lk}) \tag{7}$$

where B_{lk} is the blocking probability on link l for wavelength k. The link independence assumption is more accurate for networks where there are many alternate paths between two nodes and paths do not share many links. We feel that this assumption is sufficiently accurate for networks having a mesh topology.

A connection request that arrives on a link and finds a free wavelength does not immediately produce a new call in service. If this wavelength is not available on the rest of the path, this connection request cannot be established. Hence, the traffic offered to a link depends on the blocking probability of links that appear before and after it on a path. Let A_{lk}^p denote the offered traffic to link l originating from path p for wavelength k. A_{lk}^p is given by the so-called *reduced-load model* [16], [27] (this technique is also called the Erlang fixed-point equation)

$$A_{lk}^p = A_k^p \prod_{l' \in p, l' \neq l} (1 - B_{l'k}) = A_k^p \frac{1 - B_k^p}{1 - B_{lk}}, \qquad \text{for } l \in p \tag{8}$$

where A_k^p is the offered traffic to path p for wavelength k.

The total traffic A_{lk} offered to link l for wavelength k is given by the sum of the offered loads for the paths passing through l

$$A_{lk} = \sum_{p:\, l \in p} A_{lk}^{p}. \tag{9}$$

The link blocking probability B_{lk} for wavelength k resulting from the reduced link load A_{lk} given by (9) is found by using the equivalent random method and the Brockmeyer model given by (5). Let V_{lk} denote the variance of the traffic offered to link l for wavelength k. Since the traffic offered for wavelength 1 is Poisson, its mean is equal to its variance, i.e., $V_{l1} = A_{l1}$ for all links. The mean $\mathcal{O}_{lk}$ and variance $\hat{V}_{lk}$ of the overflow traffic from wavelength k for link l are given by

$$\mathcal{O}_{lk} = A_{lk}^{*} E(A_{lk}^{*},\, M_l + M_{lk}^{*}) \tag{10}$$

and

$$\hat{V}_{lk} = \mathcal{O}_{lk}\left(1 - \mathcal{O}_{lk} + \frac{A_{lk}^{*}}{M_l + M_{lk}^{*} + 1 - A_{lk}^{*} + \mathcal{O}_{lk}}\right). \tag{11}$$

The variance of the overflow traffic from wavelength k is equal to the variance of the offered traffic for wavelength $k+1$, i.e.,

$$V_{l,\, k+1} = \hat{V}_{lk}. \tag{12}$$

The parameters A_{lk}^{*} and M_{lk}^{*} in (10) and (11) are the equivalent Poisson traffic load and the equivalent number of fibers, respectively, which are given by the solution to the equations

$$A_{lk} = A_{lk}^{*} E(A_{lk}^{*},\, M_{lk}^{*}) \tag{13}$$

and

$$V_{lk} = A_{lk}\left(1 - A_{lk} + \frac{A_{lk}^{*}}{M_{lk}^{*} + 1 + A_{lk} - A_{lk}^{*}}\right). \tag{14}$$

The solutions A_{lk}^{*} and M_{lk}^{*} to (13) and (14) are obtained iteratively, and the link blocking probability B_{lk} is given by

$$B_{lk} = E(A_{lk}^{*},\, M_l + M_{lk}^{*}). \tag{15}$$

Note that the parameter M_{lk}^{*} is, in general, not an integer, and the generalized Erlang-B function is used in (13) and (15) [16].

Given the mean link traffic A_{lk} and variance V_{lk}, the link blocking probabilities are computed using (13)–(15), and these blocking probabilities are used to obtain the mean link traffic A_{lk} by using (8) and (9). This iterative procedure is continued until the link blocking probabilities converge. Once the procedure for wavelength k is finished, the mean and the variance of the traffic for wavelength $k+1$ are obtained from (6) and (7), and (10)–(12), respectively.

The algorithm for obtaining the blocking probability with the overflow model is described below in detail.

For $k = 1$:

1.1) compute $\{A_1^p\}$ from (4);
1.2) assume initial values for link blocking probabilities $\{B_{l1}\}$;
1.3) for each link l, find A_{l1} from (8) and (9);
1.4) for each link l, $V_{l1} = A_{l1}$;
1.5) solve for A_{l1}^{*} and M_{l1}^{*} by iterating between (13) and (14);
1.6) compute B_{l1} from (15);
1.7) if B_{l1} values converged, compute $\{B_1^p\}$ from (7), obtain $\{\mathcal{O}_{l1}\}$, $\{\hat{V}_{l1}\}$, $\{V_{l2}\}$ from (10)–(12), and go to $k = 2$; else go to 1.3.

For $1 < k \leq K$:

k.1) compute $\{A_k^p\}$ from (6);
k.2) assume initial values for link blocking probabilities $\{B_{lk}\}$;
k.3) for each link l, find A_{lk} from (8) and (9);
k.4) solve for A_{lk}^{*} and M_{lk}^{*} by iterating between (13) and (14);
k.5) compute B_{lk} from (15);
k.6) if B_{lk} values converged, compute $\{B_k^p\}$ from (7), obtain $\{\mathcal{O}_{lk}\}$, $\{\hat{V}_{lk}\}$, $\{V_{l,\,k+1}\}$ from (10)–(12), and go to $k+1$; else go to k.3.

Once $\{B_k^p,\, k = 1, \cdots, K\}$ are computed, the connection blocking probability is readily calculated. A connection request is rejected when it is not possible to establish it on any wavelength, and the mean connection blocking probability P_b is given by

$$P_b = \frac{\displaystyle\sum_p A_1^p \prod_{k=1}^{K} B_k^p}{\displaystyle\sum_p A_1^p}. \tag{16}$$

The overflow model is more accurate for fairly connected topologies for which the link independence assumption (7) is applicable. This assumption does not hold for networks with weak connectivity such as the ring topology. The accuracy of the model decreases as the number of wavelengths increases because of the successive application of the equivalent random approximation. However, the output of the overflow model matches well with the simulations for $K = 8$ wavelengths as shown in the next section.

B. Numerical Results

Although the overflow model can be applied to arbitrary mesh networks, the iterative procedure explained above for obtaining the blocking probability requires extensive computation when the numbers of links and paths in the network are large. In order to be able to apply the overflow model to a network with moderate size, we choose the symmetric mesh–torus network with 16 nodes and 32 links [10], [12]. The links in this network have the same offered traffic; hence, link blocking probabilities are identical for all wavelengths. Each link has a single fiber with $K = 8$ wavelengths. Connection requests arrive at a node according to a Poisson process with rate λ and uniformly selected destination addresses. Each established connection holds for an exponentially distributed period with unit mean.

The call blocking probability is plotted in Fig. 6 as a function of the offered load per link per wavelength which

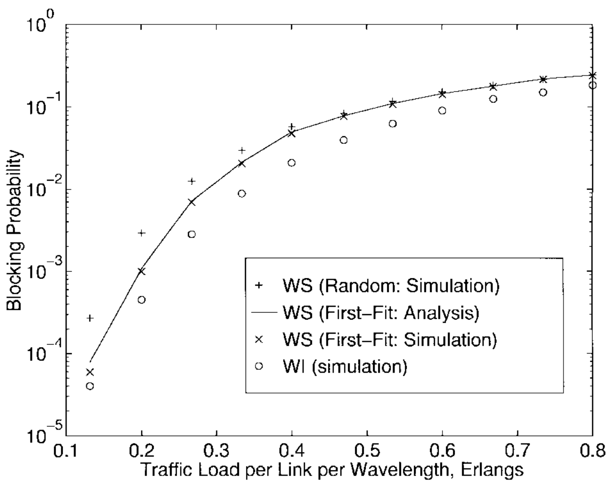

Fig. 6. Blocking probability versus link utilization for the mesh–torus network with 16 nodes.

is given in Erlangs as

$$\rho = \frac{\lambda H}{2K}$$

where $H = 32/15$ is the average path length.

Also plotted in Fig. 6 are the blocking probabilities for the first-fit and random wavelength assignment algorithms as well as the blocking probability for the WI network, all obtained from the simulations. The blocking probabilities obtained using the overflow model and from the simulations match closely although the analysis seems to overestimate the blocking probability. This difference is partly generated by the inaccuracy of the independence assumption (7). As shown in Fig. 6 through analysis and simulations, the random wavelength assignment algorithm performs poorly compared to the first-fit algorithm, especially at lower utilizations.

IV. LLR ALGORITHM

In Section II we studied the performance of the fixed shortest path routing algorithm where the physical path used by each connection is predetermined. In this section the benefits of wavelength conversion are studied for the LLR algorithm that dynamically selects paths among a set of alternative routes. Dynamic routing algorithms enable the network to adapt to changing traffic load and reduce the blocking probability. Dynamic routing algorithms have been used in circuit-switched public telephone networks since the mid-1980's (e.g., AT&T's real-time network routing (RTNR) algorithm [28], British Telecom's dynamic alternate routing (DAR) algorithm [29]).

It is expected intuitively that the wavelength conversion gain increases with the LLR algorithm primarily due to two reasons. First, the alternate paths are longer than the shortest path (large H). Second, the interference length L for a path decreases as the path set gets larger as shown in Fig. 2. These two trends result in an increased effective path length H/L, and the conversion gain is expected to increase as predicted by the Barry–Humblet model [14].

The LLR algorithm chooses the least congested path and wavelength among the available wavelengths over k shortest paths. Hence, the objective of the LLR algorithm is to reduce the blocking probability. We use simulations to compare the performance of this routing scheme with WS and WI networks.

The path and wavelength selection technique for the LLR algorithm is described as follows. Let $U_l = \sum_{j=1}^{K} A_{lj}$ for link l, where A_{lj} is the number of fibers for which wavelength j is utilized. The path set contains the set of k shortest paths between all node pairs. The least-loaded path within the path set is selected according to the following criteria. For each connection request with WI, LLR chooses the path p that achieves

$$\max_{p} \min_{l \in p} KM_l - U_l \tag{17}$$

and with WS, LLR chooses the path p and wavelength j pair that achieves

$$\max_{p,j} \min_{l \in p} M_l - A_{lj}. \tag{18}$$

It is possible that there are multiple route–wavelength pairs that maximize (17) and (18). Since the most-used rule performs better than the other network state-independent wavelength selection rules discussed in Section II, we break these ties by choosing the route-wavelength pair (p, j) such that j is the most utilized wavelength in the network. If there are tying pairs that cannot be broken with the most-used rule, the shortest such path is selected.

The worst-case complexity of the LLR algorithm is $O(k\overline{H}_k K)$, where $\overline{H}_k$ is the maximum path length in the path set with k shortest paths. The LLR algorithm requires real-time information about the utilizations of wavelengths on network links. The performance of these algorithms in optical networks depends on how fast information is transferred over the signaling network. For optical cross-connect networks where connection requests arrive at a slower rate and relatively long connection setup times are tolerable, the speed of information transfer is not that critical. We assume that utilization information for all fibers is available immediately to a central controller which is responsible for selecting the best path and wavelength for each connection request.

The performance of the LLR algorithm is also compared to the *MS routing* (MSR) algorithm, which has a path metric that computes the total utilization over each alternate path and wavelength (similar to the MS wavelength selection algorithm presented in Section II), i.e., MSR selects the route–wavelength pair that satisfies

$$\min_{p,j} \sum_{l \in p} \frac{A_{lj}}{M_l}.$$

The MSR algorithm uses the link utilization as the measure of link load, whereas the LLR algorithm calculates the amount of available resources to determine the best link–wavelength pair. In Fig. 7, the blocking probabilities for the LLR and MSR algorithms with $k = 1, 3, 5, 7$ are plotted versus the traffic load for the multifiber 30-node mesh network with $m = 0.5$, as described in Section II. As k increases, the

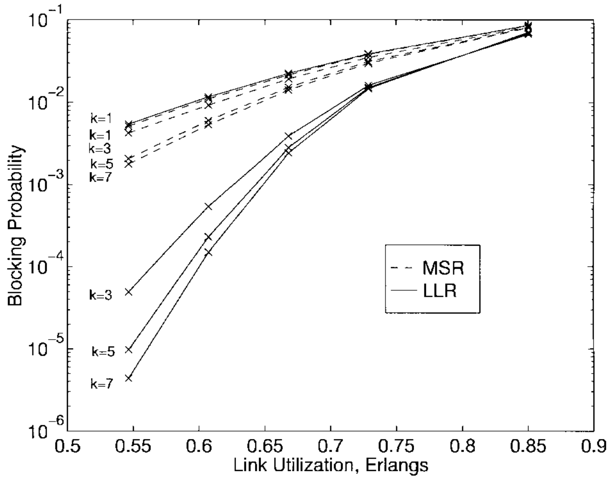

Fig. 7. Blocking probability versus link utilization for the 30-node mesh network with the LLR and MSR algorithms ($m = 0.5$).

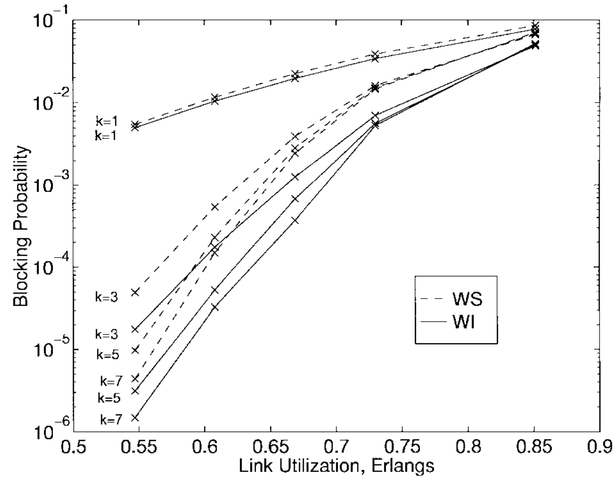

Fig. 9. Blocking probability versus network load with the LLR algorithm for $k = 1, 3, 5, 7$ ($m = 0.5$).

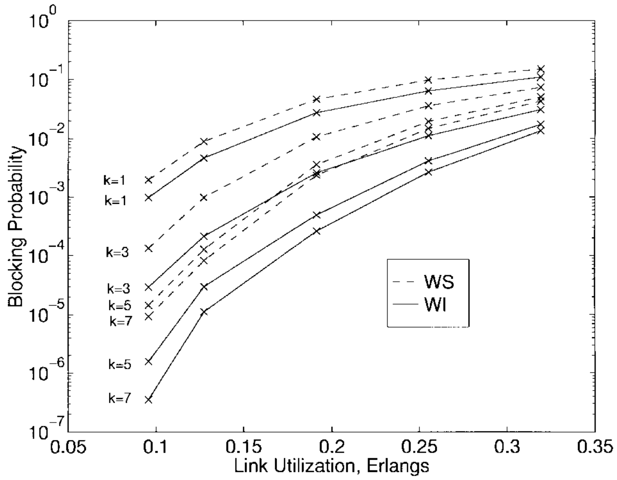

Fig. 8. Blocking probability versus network load with the LLR algorithm for $k = 1, 3, 5, 7$ (single-fiber).

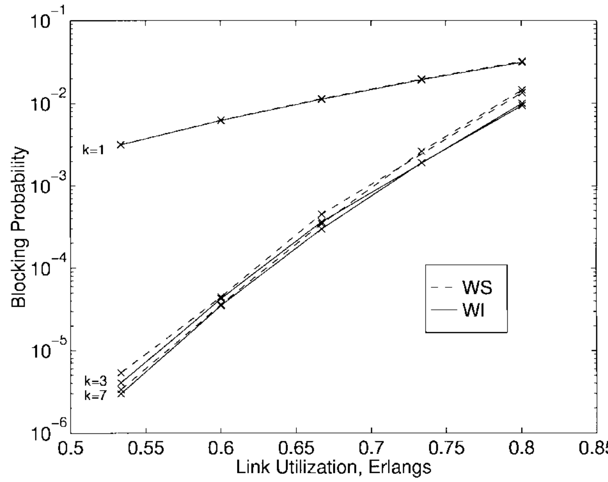

Fig. 10. Blocking probability versus network load with the LLR algorithm for $k = 1, 3, 7$ ($m = 1.0$).

blocking probabilities for both the LLR and MSR decrease, while the LLR improves at a much faster rate. Although the MSR algorithm outperforms the LLR at $k = 1$, the LLR algorithm has three orders of magnitude lower blocking probability compared to the MSR algorithm at $k = 7$ and $\rho = 0.54$.

The LLR algorithm routes more connections on alternate paths compared to the MSR algorithm, especially for connections that have a shorter distance. The average percentage of connections routed over the shortest path is 81.4% for the LLR algorithm and 91.4% for the MSR algorithm with $k = 7$ and $\rho = 0.54$. This is primarily a consequence of the summation operation in the MSR algorithm as opposed to the minimization in the LLR algorithm. As an illustration, consider a connection request between two adjacent nodes where there is a heavily loaded shortest path of one link with utilization 0.8 and a lightly loaded alternate path of three links, each with utilization of 0.3. Then the MSR algorithm prefers the heavily loaded shortest path rather than the lightly loaded alternate path. On the other hand, the load of a path for the LLR algorithm is determined by the most congested link, and the LLR algorithm chooses the less loaded alternate path (assuming all links have the same number of fibers). For the connection requests with longer paths, the difference in the use of alternate paths between two algorithms is smaller since the alternate paths are only slightly longer than the shortest path.

The blocking probabilities for $k = 1, 3, 5, 7$ are plotted for the LLR algorithm with single-fiber and multifiber ($m = 0.5, 1$) 30-node mesh network in Figs. 8–10, respectively. The performance of the LLR algorithm improves with k for both WS and WI networks. However, the rate of increase for the WI network is higher: $G_p \approx 2$ for $k = 1$ and $G_p \approx 27$ for $k = 7$ for the single-fiber network at $\rho \approx 0.1$.

On the other hand, the improvement in blocking probability with increasing k reduces with the number of fibers per link. As observed from Fig. 10, almost all of the improvement with the LLR algorithm can be achieved with $k = 3$ for $m = 1.0 (M = 9.6)$. This observation indicates that using unconstrained path sets (such as in [18]) may be unnecessary when the links comprise multiple fibers.

As shown in Fig. 11, the utilization gain G_u increases with the LLR algorithm as k increases. For the single-fiber case with

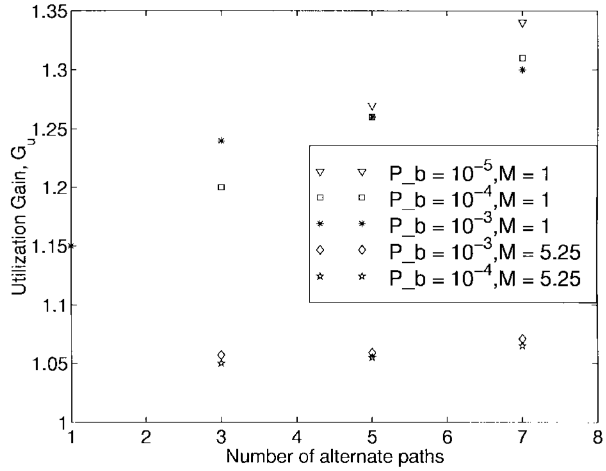

Fig. 11. G_u versus the number of alternate paths with the LLR algorithm.

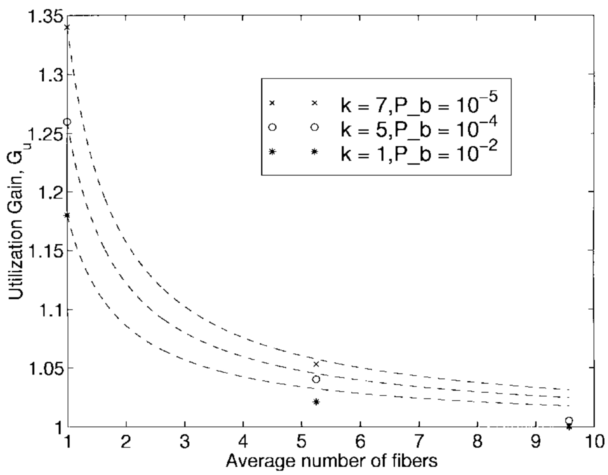

Fig. 12. G_u versus the average number of fibers for different values of k and P_b.

$P_b = 10^{-3}$, the conversion gain increases from $G_u = 1.15$ at $k = 1$ to $G_u = 1.3$ at $k = 7$. For the multifiber case, the rate of increase is smaller: $G_u = 1.05$ for $k = 3$ to $G_u = 1.07$ for $k = 7$ at $P_b = 10^{-4}$. This increase with k is largely due to longer paths (large H) and shorter interference length (small L) (as shown in Fig. 2), resulting in larger conversion gains as predicted by the Barry–Humblet model [14].

The dependence of the utilization gain G_u on the number of fibers is shown in Fig. 12. The conversion gain drops rapidly as the average number of fibers increases: $G_u = 1.34$ for the single-fiber case, and $G_u \approx 1$ for the multifiber case, where the average number of fibers per link M is 9.6 at $k = 7$ and $P_b = 10^{-5}$. The accuracy of the dependence of G_u as a function of M in the model in [13] and [15], i.e., $G_u \sim (H/L)^{(1/M)}$, is also checked in Fig. 12. The dashed curves in Fig. 12 correspond to the functions $G_1^{(1/M)}$, where G_1 is the utilization gain for $M = 1$ with corresponding values of P_b and k. We observe that this model overestimates the conversion gain, but it is fairly accurate at moderate values of M. Its accuracy diminishes as M gets larger.

V. CONCLUSIONS

In this paper we analyzed the effects of wavelength routing and selection algorithms on the wavelength conversion gain. The shortest path routing algorithm used in conjunction with a packing wavelength selection algorithm, such as the first-fit or most-used, produces a small conversion gain in our simulations with a 30-node 47-link mesh network where each link consists of a single fiber: $G_p < 2$ at $\rho \approx 0.1$ and $G_u = 1.13$ at $P_b = 10^{-3}$.

Compared to the fixed shortest path routing, the dynamic routing algorithms produce larger G_p and G_u. The LLR algorithm tries to select the path and wavelength with the minimum load, and achieves much better blocking performances for both WS and WI. Conversion gains for LLR are $G_p \approx 30$ at $\rho \approx 0.1$ and $G_u = 1.34$ at $P_b = 10^{-5}$ for a single-fiber network with $k = 7$. Although these conversion gain figures seem to favor WI networks over WS networks, these gains are obtained at very low loads for single-fiber networks which are not likely to be the case for future optical networks. As the traffic demand increases, links will have multiple fibers, and G_u decreases exponentially with the number of fibers per link, substantially reducing the gain by wavelength conversion: $G_p < 5$ at $\rho \approx 0.55$ and $G_u < 1.08$ at $P_b = 10^{-5}$ with $M = 5.25$ and $k = 7$; $G_u < 1.01$ at $P_b = 10^{-5}$ with $M = 9.6$ and $k = 7$.

We also introduced the overflow model, which is an analytical model to obtain the blocking probability for WS mesh networks employing the first-fit wavelength selection algorithm. The overflow model takes the peakedness of the overflow traffic from each wavelength into account to compute the blocking probability. The accuracy of the overflow model is good for a 16-node mesh–torus network with $K = 8$ wavelengths.

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