Analog Diversity Coding to Provide Transparent Self-Healing Communication Networks

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***Abstract*— *pg 100* Diversity coding, as introduced in [1], is a method of protection against failures in a communication network or a storage system, which is based on introducing a digital error-correcting code across independent links. This technique makes efficient use of the extra network capacity needed for coding and has the additional advantages of being nearly instantaneous, not requiring a feedback channel, rerouting, or resynchronization. In high-speed (multi Gbps) networks, digital coding will be difficult to implement, and the purpose of this paper is to demonstrate how diversity coding may be implemented in the analog domain using the Discrete Fourier Transform. In particular, we show that the DFT is a continuous-amplitude maximum-distance separable code over the field of complex numbers when the transform kernel is a prime root of unity. This code can be used to generate self-healing or fault-tolerant communication networks for continuous- or discrete-amplitude signals, as long as continuous-amplitude parity channels are available. We describe electrical and optoelectronic implementations, and a signal estimation approach to combat channel noise and thereby improve the performance of the analog diversity coding system. The most important advantage of this technique is in greatly simplifying the encoders and decoders of diversity coding systems for high-speed networks, such as fiber-optic wavelength division multiplexed networks. Application of analog diversity coding to systems with analog sources, such as telemetry systems is also possible.**

I. Introduction

Diversity CODING is an efficient method of protection against failures in a communication network or a storage system [1]. In the case of a communication network, the technique has the advantages of being nearly instantaneous, not requiring a feedback channel, rerouting, or resynchronization. In [1], we described the application of the method to digital communication networks where the encoding is performed in a finite field. In this paper, we describe the implementation of the method in the field of complex numbers to generate analog (i.e., continuous-amplitude signals). Analog diversity coding would be attractive in protecting multi Gbps transmission systems, where digital encoding will be difficult to implement. Since only the parity channels need to carry the analog signals, the data to be protected can have discrete-amplitude, and the time can be either continuous or discrete.[1]

As an example, we describe a fault-tolerant lightwave communication system: wavelength division multiplexed transmission of digital baseband signals. In this application the goal is to protect against failures in the transmitters or receivers, or disruptions in the transmission media of a subset of the channels due to polarization dispersion or other impairments.

Within the framework of information theory, diversity coding may be viewed as *coding for the erasure channel.* In [1], we showed that a generalized discrete Fourier transform (DFT), taken over a finite field, results in an *optimal* erasure channel code for binary signals when the field size is chosen appropriately, where the sense of optimality is that of minimizing the required number of redundant channels. Codes that satisfy this optimality condition are known as *maximum-distance* codes [2]. In this paper, we show how the DFT taken over the field of complex numbers can result in a maximum-distance erasure channel code for analog signals. In Section II, we describe how to use the DFT for diversity coding, and in Section III, we describe some applications and implementation techniques for analog diversity coding. Proof for the claim in Section II is given in the Appendix.

II. The Method

In this section we describe the technique of analog diversity coding. Consider Fig. 1 and let $\boldsymbol{d} = (d_1, d_2, \cdots, d_N)^t$ be a vector whose members d_k are in the field of complex numbers (to be denoted $\mathbb{C}$). This vector represents the analog data at one time instant. We are interested in forming a coded vector $\mathbf{e}$ which has $N + M$ members, such that any N members of $\mathbf{e}$ are sufficient to determine $\boldsymbol{d}$. For that purpose, we generate $\mathbf{e}$ from $\boldsymbol{d}$ via multiplication by an $(M + N) \times N$ matrix $\boldsymbol{G}$

$$\mathbf{e} = \boldsymbol{G}\boldsymbol{d} \tag{1}$$

i.e.,

$$\begin{pmatrix} e_1 \\ e_2 \\ \vdots \\ e_{N+M} \end{pmatrix} = \begin{pmatrix} \boldsymbol{g}_1 \\ \boldsymbol{g}_2 \\ \vdots \\ \boldsymbol{g}_{N+M} \end{pmatrix} \boldsymbol{d} \tag{2}$$

where $\boldsymbol{g}_k \in \mathbb{C}^N$ is the kth row vector of $\boldsymbol{G}$. Within the framework of the erasure channel model, we know which N

[1] The method is also applicable to high-resolution discrete-amplitude signals with high-resolution parity channels, enabling implementation via digital signal processing circuitry.

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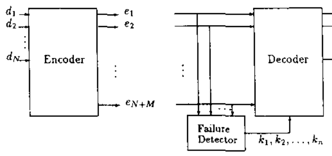
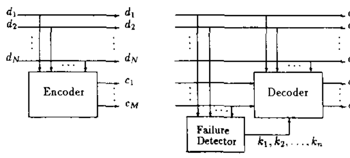


(a) Transmitter (b) Receiver

Fig. 1. The general M-for-N analog diversity coding system. The signals $d_1, d_2, \cdots, d_N$ and $e_1, e_2, \cdots, e_{N+M}$ are treated as continuous-amplitude complex numbers, and are functions of discrete or continuous time.



(a) Transmitter (b) Receiver

Fig. 2. The separable M-for-N analog diversity coding system. Note that the data channels are not encoded and therefore, during normal operation, they are untouched. Again, the signals $d_1, d_2, \cdots, d_N$ and $c_1, c_2, \cdots, c_M$ are treated as continuous-amplitude complex numbers, and are functions of discrete or continuous time.

of the links are operational. So, when $e_{k_1}, e_{k_2}, \cdots, e_{k_N}$ are available, $1 \le k_1 < k_2 < \cdots < k_N \le N+M$, we recover $\boldsymbol{d}$ via

$$\boldsymbol{d} = \begin{pmatrix} \boldsymbol{g}_{k_1} \\ \boldsymbol{g}_{k_2} \\ \vdots \\ \boldsymbol{g}_{k_N} \end{pmatrix}^{-1} \begin{pmatrix} e_{k_1} \\ e_{k_2} \\ \vdots \\ e_{k_N} \end{pmatrix}. \tag{3}$$

In order to be able to perform (3) for any set of indices $k_1, k_2, \cdots, k_N$, we require that any combination of N rows of $\boldsymbol{G}$ be linearly independent. In the case of no failures, we would like to be able to obtain $\boldsymbol{d}$ directly from $\boldsymbol{e}$, without performing any operation. In other words, we pick $e_k = d_k$ for $1 \le k \le N$. Such codes are known as *separable* codes [2]. This implies partitioning the $\boldsymbol{G}$ matrix as

$$\boldsymbol{G} = \begin{pmatrix} \boldsymbol{I} \\ \cdots \\ \boldsymbol{P} \end{pmatrix}. \tag{4}$$

Note that the system in Fig. 2 is the implementation of a separable code, where we have denoted $c_k = e_{N+k}$ for $1 \le k \le M$. In this case, the requirement that any combination of N rows of $\boldsymbol{G}$ be linearly independent is equivalent to requiring that all square submatrices of $\boldsymbol{P}$ be nonsingular.

Let $d_1, d_2, \cdots, d_N$ represent data from the lines $1, 2, \cdots, N$ respectively. We would like to protect M simultaneous line failures by providing M parity symbols $c_1, c_2, \cdots, c_M$, $1 \le M \le N$. This encoding is carried out linearly as

$$c_i = \sum_{j=1}^{N} p_{ij} d_j \qquad 1 \le i \le M \tag{5}$$

where c_i, p_{ij}, and d_j are in $\mathbb{C}$. In the notation of (4), $\boldsymbol{P} = [p_{ij}]_{M \times N}$. The parity symbols c_j are then transmitted to the receiver along with the data symbols. Consider first the case when n of the N data lines fail $(1 \le n \le M)$. At the receiver their carrier signals drop, and the receiver detects the failures. Let $k_1, k_2, \cdots, k_n$ be the indices of the links that failed; we generate signals $\tilde{c}_i$ as

$$\tilde{c}_i = c_i - \sum_{\substack{j=1 \\ j \ne k_1, k_2, \cdots, k_n}}^{N} p_{ij} d_j \qquad 1 \le i \le n. \tag{6}$$

This can easily be done since p_{ij} are fixed and known at the receiver, and d_j for $1 \le j \le N$, $j \ne k_1, k_2, \cdots, k_n$ are available. Note from (5) and (6) that

$$\tilde{c}_i = \sum_{j=k_1, k_2, \cdots, k_n} p_{ij} d_j \qquad 1 \le i \le n. \tag{7}$$

The n erased data symbols $d_{k_1}, d_{k_2}, \cdots, d_{k_n}$ can be recovered from $\tilde{c}_1, \tilde{c}_2, \cdots, \tilde{c}_n$ via an inverse linear transform, provided p_{ij} are chosen such that the row vectors $(p_{ik_1}, p_{ik_2}, \cdots, p_{ik_n})$ for $1 \le i \le n$, $1 \le k_1 < k_2 < \cdots < k_n \le N$, and $1 \le n \le M \le N$ are all linearly independent. This can be checked by considering the determinant of the matrix $\boldsymbol{B}_{k_1, k_2, \ldots, k_n} = [p_{ik_j}]_{n \times n}$.

Let

$$p_{ij} = W^{(i-1)(j-1)} \tag{8}$$

where

$$W = e^{-\hat{\imath}\frac{2\pi}{N'}}, \qquad \hat{\imath} = \sqrt{-1} \tag{9}$$

is the N'th root of unity, $W^{N'} = 1$, and we will specify N' shortly. Due to (8), we have $\boldsymbol{P}$ in (4) equal to

$$\boldsymbol{P} = \begin{pmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & W & W^2 & \cdots & W^{N-1} \\ 1 & W^2 & W^4 & \cdots & W^{2(N-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & W^{M-1} & W^{(M-1)2} & \cdots & W^{(M-1)(N-1)} \end{pmatrix} \tag{10}$$

and $\boldsymbol{B}_{k_1, k_2, \ldots, k_n}$ equal to

$$\boldsymbol{B}_{k_1, k_2, \ldots, k_n} = \begin{pmatrix} 1 & 1 & \cdots & 1 \\ W^{k_1-1} & W^{k_2-1} & \cdots & W^{k_n-1} \\ W^{2(k_1-1)} & W^{2(k_2-1)} & \cdots & W^{2(k_n-1)} \\ \vdots & \vdots & \ddots & \vdots \\ W^{(n-1)(k_1-1)} & W^{(n-1)(k_2-1)} & \cdots & W^{(n-1)(k_n-1)} \end{pmatrix}. \tag{11}$$

Note that (10) is composed of the first M rows and N columns of the DFT matrix whose number of data samples is N'. Therefore $\boldsymbol{c}$ can be calculated from $\boldsymbol{d}$ by using one of the several standard *fast Fourier transform* (FFT) methods, making use of the decomposition of multiplications to achieve parallelism, speed, and economy in circuit components.

Let

$$N' \ge N. \tag{12}$$

Then, all the elements in the second row of $\boldsymbol{P}$ in (10) are distinct, and therefore all the elements in the second row of $\boldsymbol{B}_{k_1, k_2, \ldots, k_n}$ in (11) are also distinct. Note that $\boldsymbol{B}_{k_1, k_2, \ldots, k_n}$ in (11) is a Vandermonde matrix. By using a well-known result from linear algebra, we have

$$\det \boldsymbol{B}_{k_1, k_2, \ldots, k_n} = \prod_{1 \le i < j \le n} (W^{k_j - 1} - W^{k_i - 1}). \tag{13}$$

None of the entries in the product in (13) can be zero, since W^i, $0 \le i \le N' - 1$, are *distinct* roots of unity of order N'. In other words, $W^{k_j - 1} = W^{k_i - 1}$ if and only if $i = j$. Therefore,

$$\det \boldsymbol{B}_{k_1, k_2, \ldots, k_n} \neq 0 \tag{14}$$

for $1 \le k_1 < k_2 < \cdots < k_n \le N$, $1 \le n \le M \le N$, and there exists a linear inverse transform $\boldsymbol{B}^{-1}_{k_1, k_2, \ldots, k_n}$ to obtain $d_{k_1}, d_{k_2}, \cdots, d_{k_n}$ as

$$\begin{pmatrix} d_{k_1} \\ d_{k_2} \\ \vdots \\ d_{k_n} \end{pmatrix} = \boldsymbol{B}^{-1}_{k_1, k_2, \ldots, k_n} \begin{pmatrix} \tilde{c}_1 \\ \tilde{c}_2 \\ \vdots \\ \tilde{c}_n \end{pmatrix}. \tag{15}$$

From (6) to (15) we have assumed that all the failures occur in the data lines. That is, the system is to be used to recover from $n \le M$ simultaneous line failures out of $d_1, d_2, \cdots, d_N$ in an environment where the M parity lines never fail. For this scenario, $N' \ge N$ suffices. However, we can solve the more general problem where failures are allowed in both data and parity lines by using the $\boldsymbol{P}$ matrix in (10), and by appropriately choosing the order of the root of unity. Let $c_1, c_2, \cdots, c_M$ be generated as in (5) where $p_{ij} = W^{(i-1)(j-1)}$, $W = e^{-\hat{\imath}\frac{2\pi}{N'}}$, $\hat{\imath} = \sqrt{-1}$, and N' will be specified shortly. We now assume that up to a total of M lines out of the data lines $d_1, d_2, \cdots, d_N$ and parity lines $c_1, c_2, \cdots, c_M$ can fail. Let there be $n \le M$ failures in the data lines, and let there be $m \le M - n$ failures among the M parity lines. At least n parity lines have not failed and they can be used for recovering the n failed data lines. Let us denote the n failed lines by $d_{k_1}, d_{k_2}, \cdots, d_{k_n}$ and any n of the healthy parity lines by $c_{l_1}, c_{l_2}, \cdots, c_{l_n}$, where $1 \le n \le M$, $1 \le k_1 < k_2 < \cdots < k_n \le N$, and $1 \le l_1 < l_2 < \cdots < l_n \le M$. Similarly to (6), generate signals $\tilde{c}_i$

$$\tilde{c}_i = c_{l_i} - \sum_{\substack{j=1 \\ j \neq k_1, k_2, \cdots, k_n}}^{N} p_{l_i j} d_j \tag{16}$$

$$= \sum_{j = k_1, k_2, \cdots, k_n} p_{l_i j} d_j$$

$$= \sum_{j = k_1, k_2, \cdots, k_n} W^{(l_i - 1)(j-1)} d_j \qquad 1 \le i \le n. \tag{17}$$

In other words, by using the $N - n$ healthy data lines and n healthy parity lines, we can generate n data symbols $\tilde{c}_1, \tilde{c}_2, \cdots, \tilde{c}_n$ which are linear combinations of the n failed symbols $d_{k_1}, d_{k_2}, \cdots, d_{k_n}$. We can recover the erased symbols via an inverse transform provided that the transformation matrix $\boldsymbol{B}_{k_1, k_2, \ldots, k_n; l_1, l_2, \ldots, l_n} = [W^{(l_i - 1)(k_j - 1)}]_{n \times n}$, or

$$\boldsymbol{B}_{k_1, k_2, \ldots, k_n; l_1, l_2, \ldots, l_n} = \begin{pmatrix} W^{(l_1-1)(k_1-1)} & W^{(l_1-1)(k_2-1)} & \ldots & W^{(l_1-1)(k_n-1)} \\ W^{(l_2-1)(k_1-1)} & W^{(l_2-1)(k_2-1)} & \ldots & W^{(l_2-1)(k_n-1)} \\ \vdots & \vdots & \ddots & \vdots \\ W^{(l_n-1)(k_1-1)} & W^{(l_n-1)(k_2-1)} & \ldots & W^{(l_n-1)(k_n-1)} \end{pmatrix} \tag{18}$$

linking $d_{k_1}, d_{k_2}, \cdots, d_{k_n}$ to $\tilde{c}_1, \tilde{c}_2, \cdots, \tilde{c}_n$ is invertible. Note that we would like *any* $n \times n$ square submatrix of $\boldsymbol{P}$ to be invertible, where $1 \le n \le M$. $\boldsymbol{B}_{k_1, k_2, \ldots, k_n; l_1, l_2, \ldots, l_n}$ is not a Vandermonde matrix in general. Therefore, it cannot be verified nonsingular with the method we used for $\boldsymbol{B}_{k_1, k_2, \ldots, k_n}$. However, when

$$N' \ge N, \qquad N' \text{ is prime} \tag{19}$$

then we have

$$\det \boldsymbol{B}_{k_1, k_2, \ldots, k_n; l_1, l_2, \ldots, l} \neq 0. \tag{20}$$

Based on some numerical evidence, this result was conjectured by the authors, and was recently proved by R. Stanley of MIT [3]. For completeness, we provide this proof in the Appendix.

Again, the encoding can be accomplished by using the FFT methods. There are published FFT algorithms for the case when the sample size is prime [4].

Since the DFT operates over complex variables and generates complex variables, whereas real-life data are real, a mapping needs to be made from real-valued data into complex-valued data in order to use our methods. This can be accomplished for discrete-time systems by considering one of the two consecutive samples as the real part and the other as the imaginary part of a complex number. Or, for all systems, a complex-valued channel can be formed by treating two parallel real-valued channels as one complex-valued channel. Using the first scheme, recovery from up to M failures can be achieved, and with the second scheme, up to $M/2$ pairs of failures can be restored.

III. Applications

Our method is applicable to a wide variety of problems where some redundancy is desired to protect failures or losses in systems in which analog channels are used for the transmission of coded data, where data are processed in parallel (in a space, time, or frequency sense), and where the channels in which failures can be identified via some external mechanism, such as loss of carrier or synchronization. Note that only parity channels need to carry analog data, and protected channels can carry either analog or digital data waveforms. The reason for this is that the digital waveforms are a special case of analog waveforms, and therefore, the method for analog waveforms covers digital waveforms. Depending on the particular application and implementation, time could be either discrete or continuous.

One could think of several such applications. For example, consider a deep space probe that sends measurements from the

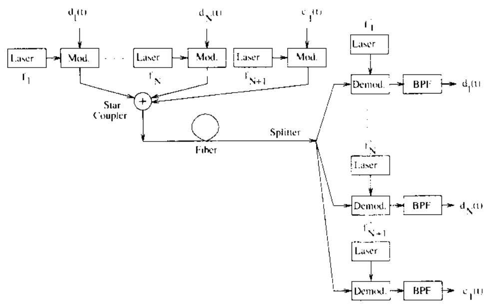


Fig. 3. Lightwave WDM transmission of digital baseband signals.

space to the earth. Some such probes use analog pulse position modulation to send analog baseband data with the possibility of more than one such telemetry channels being operational at one time. However, during operation, galactic and atmospheric noise interfere with the incoming signal, and sometimes the receiver cannot detect the incoming pulse within a time slot, in which case it declares an erasure. More importantly, sometimes one of the transmitters fails. Although, in such a situation, the earth station can identify the failed channel and direct the satellite to switch the failed channel data to a spare transmitter, the round trip propagation delay involved may cause important information to be missed. To prevent this from happening, spare channels can continuously transmit encoded data where the encoding is performed, using the methods introduced in the previous section. Then, in the case of an erasure due to either noise or equipment failure, the receiver can extract the erased data via decoding, using the information in the healthy data and parity channels. Other variations of this problem are in the transmission of analog measurement, control, and instrumentation data in the factory environment where failures, due to interference or physical disruptions, may occur; or parallel communication channels that can undergo noise, fading, interference, or jamming such as in analog secure voice systems operating on HF transmission bands over the air. Similarly, analog recordings may be protected by using our DFT methods against loss, destruction, noise, or equipment failure.

A different class of applications is for high-speed (several Gbps) transmission systems that may be transmitting digital data, but their speed precludes efficient digital implementation of diversity coding. Consider the application of our method to the wavelength division multiplexed (WDM) lightwave communication system shown in Fig. 3 [5]. In this case, the baseband signals are digital, and each modulates a laser. The generated lightwave signals are combined optically using a star coupler at the transmitter. At the receiver, the heterodyning is performed optically, and bandpass filtering and signal detection are carried out electrically. In [1], we proposed the use of finite field erasure channel codes for transmitter or receiver failures, and polarization dispersion for this application. Here, we propose the use of analog codes for protection to avoid high-speed (multi Gbps) digital circuitry. In Fig. 3, the signals $d_1(t), \cdots, d_N(t)$ that modulate

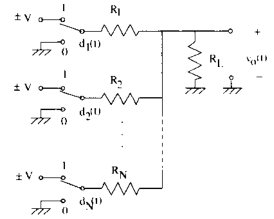


Fig. 4. Electrical implementation of the analog encoder of the binary signals $d_1(t), \cdots, d_N(t)$ for the WDM example.

lasers at frequencies $f_1, \cdots, f_N$ are digital, whereas the signal $c_1(t) = d_1(t) + \cdots + d_N(t)$ that modulates the laser at frequency f_{N+1} is discrete-valued, but we treat it as analog. In the case of a failure in one of the data channels, say $d_1(t)$, the failed channel can be recovered at the receiver as $c_1(t) - d_2(t) - \cdots - d_N(t)$. This is an interesting example where our analog diversity codes are used to protect discrete-time binary signals by generating a discrete-amplitude, continuous-time signal. Note that we can generalize this point-to-point topology into a multipoint-to-multipoint topology as in [1].

We present two implementations of analog coders that realize the linear combination of binary baseband signals. The first is an electrical implementation, and is shown in Fig. 4. The switches are actuated by the binary signals $d_1(t), \cdots, d_N(t)$ that can be either 0 or 1. The output $v_o(t)$ is given as

$$v_o(t) = VR_L\left(\pm\frac{d_1(t)}{R_1} \pm \cdots \pm \frac{d_N(t)}{R_N}\right). \tag{21}$$

By appropriately choosing V, R_L, $R_1, \cdots, R_N$, and whether to use positive or negative polarization for the constant voltage sources with magnitude V, any real combination of $d_1(t), \cdots, d_N(t)$ can be formed. Note that at the transmitter the resistors $R_1, \cdots, R_N$ are fixed since they are used for encoding. At the receiver, they should change value according to which channels have failed. This functionality can be realized by using voltage controlled resistors.

The second implementation of the analog combiner uses *optoelectronic* technology and is shown in Fig. 5. In this implementation, signal sources $d_1(t), \cdots, d_N(t)$ drive an array of lasers, each signal modulating one laser. The lasers illuminate a mask whose opaqueness is varied, according to the linear combination coefficients in the transform. Using focusing optics, the light through the mask is collected on a photodiode, whose output signal is a linear combination of $d_1(t), \cdots, d_N(t)$ where the coefficients are determined by the mask. Note that this method restricts the coefficients to positive real numbers. To realize both positive and negative coefficients, two such photodiodes and two rows in the mask are used. One row and one photodiode are dedicated to the positive coefficients in the linear combination, and the other to the negative coefficients. Then, the outputs of the two photodiodes are subtracted from each other to form the desired real linear combination of the signal sources. Thus, for a complex number, four rows and four photodiodes are needed. By stacking up photodiodes and rows of the interconnectivity mask, the product of the signal vector with a complex matrix

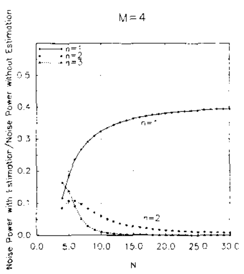


Fig. 5. Optoelectronic implementation of the analog encoder for the WDM example. In this figure, $\mathrm{Re}[c_1(t)] = a_1 d_1(t) - a_2 d_2(t) = a_3 d_3(t)$. $\mathrm{Im}[c_1(t)]$ and the real and imaginary components of $c_j(t)$ for $2 \leq j \leq M$ are obtained similarly.

can easily be computed. The mask to be used at the transmitter can be obtained using a fixed film. For the receiver, spatial light modulators whose opaqueness can be varied via control signals (which are not shown) can be used. This method is used in optical neural computers to implement adaptive weights in conjunction with a learning mechanism such as the back-propagation algorithm [6]. The advantage of this technique is the ability to perform large matrix products easily, fast, and at high frequencies without running into inductive or capacitive problems as might be the case for the technique in Fig. 4.

IV. Signal Estimation for Analog Diversity Coding

Since the decoding operation correlates noise belonging to different channels, the analog diversity coding scheme in the previous sections suffers from noise enhancement when the transmission medium is noisy. We now describe a signal estimation approach to combat channel noise and thereby improve the performance of the analog diversity coding system. With this method, the linear operation performed at the decoder involves all of the healthy data and parity channels, as opposed to N healthy channels, where N is the number of information-bearing channels, as in the scheme in the previous sections. This approach provides an averaging of the noise, reducing the effective noise power on each recovered channel.

When there is noise in the channel, the recovery via (3) or via (15) results in noise enhancement. For example, for $M = 1$, and when a data channel fails, say k_1, it is recovered as

$$\hat{d}_{k_1} = \sum_{k=1}^{N} d_k + n_0 - \sum_{\substack{k=1 \\ k \neq k_1}}^{N} (d_k + n_k) = d_{k_1} - \sum_{\substack{k=0 \\ k \neq k_1}}^{N} n_k \quad (22)$$

where n_k is the noise on the kth channel, $1 \leq k \leq N$, and n_0 is the noise on the protection channel. When the noise is distributed independently and identically with zero mean and variance σ_n^2 from channel to channel, the output noise power in the recovered link k_1 will be equal to $N\sigma_n^2$, a large increase from σ_n^2. In general, under the independence and the identical distribution assumption, the average noise in the recovered links will be

$$\frac{1}{N}\sigma_{\hat{d}-d}^2 = \frac{1}{N} E(\hat{\boldsymbol{d}} - \boldsymbol{d})^H (\hat{\boldsymbol{d}} - \boldsymbol{d})$$
$$= \frac{1}{N}\sigma_n^2 \operatorname{tr}[(\boldsymbol{G}_{k_1, k_2, \ldots, k_n}^{-1})(\boldsymbol{G}_{k_1, k_2, \ldots, k_n}^{-1})^H] \quad (23)$$

where we defined

$$\boldsymbol{G}_{k_1, k_2, \ldots, k_n} = \begin{pmatrix} \boldsymbol{g}_{k_1} \\ \boldsymbol{g}_{k_2} \\ \vdots \\ \boldsymbol{g}_{k_n} \end{pmatrix}, \quad (24)$$

and used H to represent the complex conjugate transpose, and tr the trace of a matrix. $\hat{\boldsymbol{d}}$ is the right hand side of (3), which in the presence of channel noise is not equal to $\boldsymbol{d}$.

When $n \leq M$ link failures occur, the approach in Section II uses exactly n of the parity channels for recovery. In the case of no noise, data in the remaining $M - n$ channels are redundant, and can be ignored. When there is noise, however, data in these channels can be used to reduce the noise power and thereby enhance the signal. Assume that $n \leq M$ lines have failed, and the links $k_1, k_2, \cdots, k_{N+M-n}$ are active. The received vector is

$$\boldsymbol{e} = \boldsymbol{G}_{k_1, k_2, \ldots, k_{N+M-n}} \boldsymbol{d} + \boldsymbol{n} \quad (25)$$

where $\boldsymbol{e}$ and $\boldsymbol{n}$ have $N + M - n$ members, and $\boldsymbol{G}_{k_1, k_2, \ldots, k_{N+M-n}}$ is as defined in (24). Based on $\boldsymbol{e}$, we want to recover $\boldsymbol{d}$, i.e., choose a $\hat{\boldsymbol{d}}$ optimally close to $\boldsymbol{d}$ in some sense. The criterion of closeness, that is, on which sense we choose $\hat{\boldsymbol{d}}$ determines the decoder.

A. Least Squares Estimation

First, consider estimation of $\boldsymbol{d}$ from $\boldsymbol{e}$ in the *least squares* sense. In this case, we require that the estimate minimize

$$\|\boldsymbol{G}_{k_1, k_2, \ldots, k_{N+M-n}} \hat{\boldsymbol{d}} - \boldsymbol{e}\|^2$$
$$= (\boldsymbol{G}_{k_1, k_2, \ldots, k_{N+M-n}} \hat{\boldsymbol{d}} - \boldsymbol{e})^H (\boldsymbol{G}_{k_1, k_2, \ldots, k_{N+M-n}} \hat{\boldsymbol{d}} - \boldsymbol{e}). \quad (26)$$

Note the *absence* of the expectation operator in (26). In fact, we do not assume we know any statistics about the problem. We only assume that we have some observation $\boldsymbol{e}$, and we fit some data $\boldsymbol{d}$ to it such that when $\boldsymbol{d}$ is transformed via $\boldsymbol{G}_{k_1, k_2, \ldots, k_{N+M-n}}$ it is close to our observation $\boldsymbol{e}$. By straightforward minimization with respect to $\hat{\boldsymbol{d}}$, (26) yields the optimum estimate in the least squares sense:

$$\hat{\boldsymbol{d}} = (\boldsymbol{G}_{k_1, k_2, \ldots, k_{N+M-n}}^H \boldsymbol{G}_{k_1, k_2, \ldots, k_{N+M-N}})^{-1} \cdot \boldsymbol{G}_{k_1, k_2, \ldots, k_{N+M-n}}^H \boldsymbol{e} \quad (27)$$

which is equal to the well-known least squares solution for the noisy observations of an overdetermined set of linear equations. For the method to work for any $n \leq M$ failures, the inverse in (27) should always exist. It is well-known that $\boldsymbol{A}^H \boldsymbol{A}$ is nonsingular if the columns of $\boldsymbol{A}$ are linearly independent [7]. It has been proved in [3] that any N rows of $\boldsymbol{G}_{k_1, k_2, \ldots, k_{N+M-n}}$ are linearly independent, in other words that $\boldsymbol{G}_{k_1, k_2, \ldots, k_{N+M-n}}$ is of full rank. Therefore, the columns of $\boldsymbol{G}_{k_1, k_2, \ldots, k_{N+M-n}}$ are linearly independent, consecutively the matrix $G^H_{k_1,k_2,\ldots,k_{N+M-n}} G_{k_1,k_2,\ldots,k_{N+M-n}}$ is always nonsingular for any $1 \le k_1 < k_2 < \cdots < k_{N+M-n} \le N+M$, $n \le M$, and the estimate in (27) is well-defined. In this case, when the noise is independently and identically distributed with zero mean and variance σ_n^2 from channel to channel, the average noise after estimation is

$$\begin{aligned}\frac{1}{N}\sigma^2_{\hat{d}-d} &= \frac{1}{N}E(\hat{\boldsymbol{d}}-\boldsymbol{d})^H(\hat{\boldsymbol{d}}-\boldsymbol{d}) \\ &= \frac{1}{N}\sigma_n^2 \operatorname{tr}[(G^H_{k_1,k_2,\ldots,k_{N+M-n}} G_{k_1,k_2,\ldots,k_{N+M-n}})^{-1}]. \end{aligned} \quad (28)$$

Note that when $\boldsymbol{n}$ is zero-mean, the least squares estimate is unbiased (i.e. $(E\hat{\boldsymbol{d}} = E\boldsymbol{d})$.

The method can be used to enhance the signal even when there are no link failures, that is, when $n = 0$. In this case, due to (4), $G^H G$ can be decomposed as

$$G^H G = I + P^H P. \quad (29)$$

This matrix can be inverted using the matrix inversion lemma and noting that

$$PP^H = NI \quad (30)$$

since the rows of P are a set of orthogonal basis vectors for $\mathbb{C}^N$, which yields

$$(G^H G)^{-1} = I - \frac{1}{N+1}P^H P. \quad (31)$$

On the other hand, the ith row, jth column entry of $P^H P$ can be calculated as

$$(P^H P)_{ij} = \sum_{k=0}^{M-1} W^{(j-i)k} = \frac{1 - W^{(j-i)M}}{1 - W^{j-i}}. \quad (32)$$

Therefore, (28) becomes

$$\frac{1}{N}\sigma^2_{\hat{d}-d} = \sigma_n^2\left(1 - \frac{M}{N+1}\right). \quad (33)$$

Equation (33) shows that using the signal estimation in (27), the reduction in noise power increases linearly with M for $1 \le M \le N$. If $M = N$, by transmitting every d_k on two separate channels and averaging the received signals one can reduce the noise power in each data channel by a factor of one half. On the other hand, note from (33) that when $M = N$, the method of (27) results in a noise power reduction of $(N+1)$-fold.

When there are link failures, that is, when $n > 0$, there will be some reduction in noise power similar to (33). However, an analytical calculation of this reduction is difficult. It can be numerically calculated as the ratio of (28) to (23). In Fig. 6, we plot the results of this calculation for $1 \le N \le 30$, $M = 4$, and $1 \le n \le M-1$. For $n = M$, (28) reduces to (23), and the two methods result in the same decoding matrix, and therefore, the same noise power.

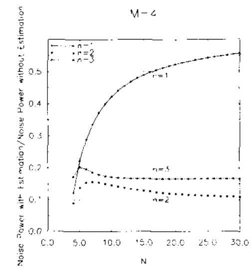


Fig. 6. Reduction in noise power after estimation using the least squares estimate for $M = 4$.

B. "Best" Linear Unbiased Estimate

Note that the least squares estimate in (27) does not require any knowledge of the signal or the noise statistics. The availability of these statistics improves the estimate. In particular, when the constraint that the estimate be unbiased and efficient, i.e., that the error variance in each of the members of $\hat{\boldsymbol{d}}$ is minimum is imposed, one obtains the so-called *best linear unbiased estimate* [8]

$$\begin{aligned}\hat{\boldsymbol{d}} = (G^H_{k_1,k_2,\ldots,k_{N+M-n}} R_n^{-1} G_{k_1,k_2,\ldots,k_{N+M-n}})^{-1} \\ \cdot G^H_{k_1,k_2,\ldots,k_{N+M-n}} R_n^{-1} e \end{aligned} \quad (34)$$

where R_n is the autocorrelation matrix of the noise, $R_n = E\boldsymbol{n}\boldsymbol{n}^T$. Note that only noise statistics, and not the data statistics are needed in (34). For identically and independently distributed noise, the estimates in (27) and (34) are the same. Otherwise, the estimate in (34) has the property of weighting the contributions of samples with small noise power heavily and those of large power lightly.

Note that in the term "best linear unbiased estimate," "best" is in the sense of minimizing the error norm when the signal statistics are not known. When the signal statistics are known, one can obtain better error norm performance as described below.

C. Minimum Mean Squared Error Estimation

When the signal has zero mean and its autocorrelation matrix $R_d = E\boldsymbol{d}\boldsymbol{d}^T$ is known, one can write

$$\begin{pmatrix} e \\ \cdots \\ 0 \end{pmatrix} = \begin{pmatrix} G_{k_1,k_2,\ldots,k_{N+M-n}} \\ \cdots\cdots\cdots \\ I \end{pmatrix}\boldsymbol{d} + \begin{pmatrix} \boldsymbol{n} \\ \cdots \\ -\boldsymbol{d} \end{pmatrix}. \quad (35)$$

By using the result for the best linear unbiased estimate (34), the estimate for $\boldsymbol{d}$ in (35) can be obtained as [8]:

$$\begin{aligned}\hat{\boldsymbol{d}} = (R_d^{-1} + G^H_{k_1,k_2,\ldots,k_{N+M-n}} R_n^{-1} G_{k_1,k_2,\ldots,k_{N+M-n}})^{-1} \\ \cdot G^H_{k_1,k_2,\ldots,k_{N+M-n}} R_n^{-1} e. \end{aligned} \quad (36)$$

This becomes, via the matrix inversion lemma

$$\hat{\boldsymbol{d}} = R_d \overline{G}^H (I - (\overline{G}^H R_d \overline{G} + R_n)^{-1} \overline{G}^H R_d \overline{G}) R_n^{-1} e \quad (37)$$

$$= R_d G^H_{k_1,k_2,\ldots,k_{N+M-n}} (G_{k_1,k_2,\ldots,k_{N+M-n}} R_d G_{k_1,k_2,\ldots,k_{N+M-n}} + R_n)^{-1} e \quad (38)$$

where we have used $\overline{G}$ for $G_{k_1, k_2, \ldots, k_{N+M-n}}$ in (37). Equation (38) is the Wiener solution, i.e., the estimate that minimizes $E\|\hat{d} - d\|^2$, equal to

$$\hat{\boldsymbol{d}} = E(\boldsymbol{d}\boldsymbol{e}^H)[E(\boldsymbol{e}\boldsymbol{e}^H)]^{-1} \tag{39}$$

also known as the *minimum variance estimate*, or the *minimum mean squared error estimate*.

We now would like to calculate the noise power after estimation in the minimum mean squared error estimate when there are no link failures, similarly to (33). Assuming that the signal has zero mean and the autocorrelation matrix $\boldsymbol{R}_d = \sigma_d^2 \boldsymbol{I}$, i.e., statistically independent from channel to channel, and the noise has zero mean and the autocorrelation matrix $\boldsymbol{R}_n = \sigma_n^2 \boldsymbol{I}$, i.e., again, statistically independent from channel to channel, (36) becomes

$$\hat{\boldsymbol{d}} = \frac{\sigma_D^2}{\sigma_d^2 + \sigma_n^2}\left(\boldsymbol{I} + \frac{\sigma_d^2}{\sigma_d^2 + \sigma_n^2}\boldsymbol{P}^H\boldsymbol{P}\right)^{-1}\boldsymbol{G}^H\boldsymbol{e} \tag{40}$$

$$= \frac{\sigma_d^2}{\sigma_d^2 + \sigma_n^2}\left(\boldsymbol{I} - \frac{\sigma_d^2}{(N+1)\sigma_d^2 + \sigma_n^2}\boldsymbol{P}^H\boldsymbol{P}\right)\boldsymbol{G}^H\boldsymbol{e}. \tag{41}$$

Using this result, (30), and (32), after some manipulation, one obtains

$$\frac{1}{N}\sigma_{\hat{\boldsymbol{d}}-\boldsymbol{d}}^2 = \sigma_n^2\left(\frac{\frac{\sigma_d^2}{\sigma_n^2}}{1+\frac{\sigma_d^2}{\sigma_n^2}}\right)^2\left(1 + \frac{M\left(1 - \frac{\sigma_d^4}{\sigma_n^4}(N+1)\right)}{\left(1+\frac{\sigma_d^2}{\sigma_n^2}(N+1)\right)^2}\right). \tag{42}$$

This result should be contrasted with (33), to which it reduces when the signal-to-noise ratio $SNR = \sigma_d^2/\sigma_n^2$ goes to infinity. By plotting (42), it can be seen that the noise power reduction saturates around an SNR of 10 or 20. For larger SNR values, the performance with the minimum mean squared error estimate is the same as that with the least square estimate, that is, there is no improvement gained by knowing the signal statistics. For smaller values of SNR, however, the estimate is weighted approximately by a factor of $(1 + SNR^{-1})^{-1}$, which reduces the noise power by estimation approximately by the square of this factor.

As in the case of least squares estimation, an analytical calculation of the reduction in noise power after estimation is difficult in the case of minimum mean squared error estimation. We provide a numerical evaluation, and plot the results in Fig. 7. We assume the noise is independent from channel to channel, has zero mean and variance σ_n^2; likewise, we assume the signal is independent from channel to channel, has zero mean and variance σ_d^2. The number of data channels N varies between 1 and 30, and the number of parity channels M is equal to 4. In this case, the reduction in noise power is much more than in the case of least squares estimation. For example, for $N = 10$, $M = 4$ and $n = 1$, the reductions in noise power are 0.42 and 0.32 with least squares estimation and minimum mean squared estimation, respectively. On the other hand, for $N = 10$, $M = 4$ and $n = 3$, the reductions in noise power are 0.17 and 0.0011 (i.e., a factor of 20 dB) with least squares estimation and minimum mean squared estimation, respectively.

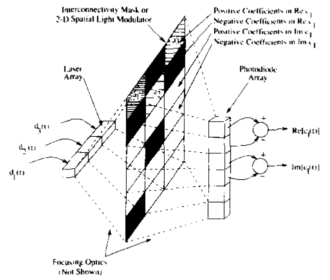


Fig. 7. Reduction in noise power after estimation using the minimum mean squared error estimate for $M = 4$.

V. Summary and Conclusions

We described the *optimal* use of the DFT over the field of complex numbers (in the sense of minimizing the number of extra channels required), to perform analog diversity coding. In particular, we have shown that the DFT can be used to generate a continuous-amplitude *maximum-distance separable* code over the field of complex numbers when the transform kernel is a prime root of unity. This coding can be used to generate fault-tolerant communication networks for continuous- or discrete-amplitude signals, as long as continuous-amplitude parity channels are available. We have described electrical and optoelectronic implementations of this code. The most important advantage of this technique is in greatly simplifying the encoders and decoders of diversity coding systems for high-speed networks, such as fiber-optic WDM networks. Furthermore, application of analog diversity coding to systems with analog sources, such as telemetry systems is also possible.

We showed how straightforward signal estimation can enhance the performance of analog diversity coding systems in the presence of channel noise by making use of the information in the remaining $M - n$ channels when n link failures occur in an analog diversity coding system of N data channels. With least squares estimation, and with an added complexity by a factor of M/N multiply/add operations, the noise power after estimation is reduced by about 3 dB for $n = 1$, or by about 10 dB or more for $1 < n < M$, as compared to the case without estimation. This approach does not require the statistics of the signal or the noise be known. When the second order statistics of the signal and the noise are known, the minimum mean squared error estimation can be used to reduce the noise power by more than 3 dB for $n = 1$, and by about 13 dB or more for $1 < n < M$ as compared to the case without estimation, without any added complexity. In particular, for $n > 1$, the ratio of noise power with and without estimation is in the negative tens of dB's as N gets large.

Acknowledgment

We gratefully acknowledge discussions with H. Landau and N. J. A. Sloane concerning our conjecture in the claim.

Appendix

Claim: Let $\zeta = e^{i\frac{2\pi}{P}}$, P a prime integer, and let $p_1, p_2, \cdots, p_N$ and $q_1, q_2, \cdots, q_N$ be integers in the range

$[0, P-1]$ with $p_m \neq p_n$, $q_m \neq q_n$ for $m \neq n$. Then the matrix, whose (j, k)th entry is $\zeta^{p_j q_k}$, $1 \leq j, k \leq N$, is nonsingular.

Proof (due to R. Stanley [3]): Let D denote the determinant $\det[\zeta^{p_j q_k}]$, $1 \leq j, k \leq N$. Consider the determinant $\det[z_j^{q_k}]$, $1 \leq j, k \leq N$. This is a polynomial in the variables $z_1, z_2, \cdots, z_N$, which vanishes whenever $z_m = z_n$, since then two rows of the determinant coincide. Consequently, that determinant has a factor the polynomial $\prod_{j<k}(z_k - z_j)$, $1 \leq j, k \leq N$, and the quotient is a polynomial in $z_1, z_2, \cdots, z_N$ with integer coefficients. To see this explicitly, we can subtract the top row from each of the others, and extract a factor $z_k - z_1$ from the kth row. This yields as quotient the determinant whose top row is $z^{q_1}, z^{q_2}, \cdots, z^{q_N}$ and whose kth row is $(z_k^{q_1} - z_1^{q_1})/(z_k - z_1)$, a polynomial with integer coefficients. In this determinant, we now subtract the second row from each subsequent row, and extract factors $(z_3 - z_2) \cdots (z_N - z_2)$. Continuing this procedure, we find

$$\frac{\det[z_j^{q_k}]}{\prod_{j<k}(z_k - z_j)} = \sum c_\alpha z_1^{\alpha_1} z_2^{\alpha_2} \cdots z_N^{\alpha_N} \tag{43}$$

with α denoting the multi-index $(\alpha_1, \alpha_2, \cdots, \alpha_N)$, and c_α an integer, as was desired. We can evaluate $\sum c_\alpha$ in (43) by substituting $z_j = w^j$, $1 \leq j \leq N$, whereupon the numerator on the left becomes $\det[(w^j)^{q_k}] = \det[(w^{q_k})^j]$, a Vandermonde determinant equal to $w^{q_1+q_2+\cdots+q_N}\prod_{j<k}(w^{q_k} - w^{q_j})$. On letting $w \to 1$, we thus find from (43)

$$\sum c_\alpha = \lim_{w \to 1} \prod_{j<k} \frac{w^{q_k} - w^{q_j}}{w^k - w^j} = \prod_{j<k} \frac{q_k - q_j}{k - j}.$$

On setting $z_j = \zeta^{p_j}$, $1 \leq j \leq N$, in (43) becomes $D/\prod_{j<k}(\zeta^{p_j} - \zeta^{p_k})$. We will now derive a contradiction from the vanishing of D. For as the denominator does not vanish, we see that if $D = 0$, then

$$\sum c_\alpha \zeta^{p_1\alpha_1 + p_2\alpha_2 + \cdots + p_N\alpha_N} = 0. \tag{44}$$

Consider now $T(w) = \sum c_\alpha \zeta^{p_1\alpha_1 + p_2\alpha_2 + \cdots + p_N\alpha_N}$, a polynomial in w with integer coefficients, which, by (44) vanishes at ζ. Since

$$L(w) = w^{P-1} + w^{P-2} + \cdots + w + 1 \tag{45}$$

is the polynomial of least degree over the integers vanishing at ζ, it must be a factor of $T(w)$, and so

$$T(w) = Q(w)L(w) \tag{46}$$

where $Q(w)$ is a polynomial with integer coefficients. On evaluating at $w = 1$ we find

$$\sum c_\alpha = T(1) = Q(1)P. \tag{47}$$

but this cannot happen, since, by (44), all the factors are in the range $[-(P-1), (P-1)]$. This contradiction shows that $D \neq 0$, and so establishes the Proposition.

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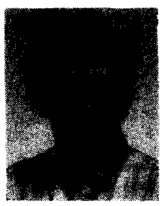

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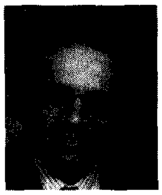

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Dr. Gitlin is the author of more than 50 technical papers, numerous conference papers, and he holds 25 patents in the areas of data communications, digital signal processing, and broadband networking. He is a co-author of the forthcoming text, *Data Communication Principles*. He is co-author of a paper on fractionally spaced adaptive equalization that was selected as the Best Paper in Communications by the *Bell System Technical Journal* in 1982. He is a member of Sigma Xi, Tau Beta Pi, and Eta Kappa Nu. He has served as chairman of the Communication Theory Committee of the IEEE Communications Society, as a member of the COMSOC Awards Board, Editor for Communication Theory of the IEEE TRANSACTIONS ON COMMUNICATIONS, and a member of the Editorial Advisory Board of the PROCEEDINGS OF THE IEEE. Currently, he is a member of the Board of Governors of the IEEE Communications Society.

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Israel Bar-David (A'56–M'60–SM'78–F'80) for a photograph and biography, please see the July 1991 issue of this TRANSACTIONS, p. 1064.