A Simple Optimization Model for Wireless Opportunistic Routing with Intra-session Network Coding

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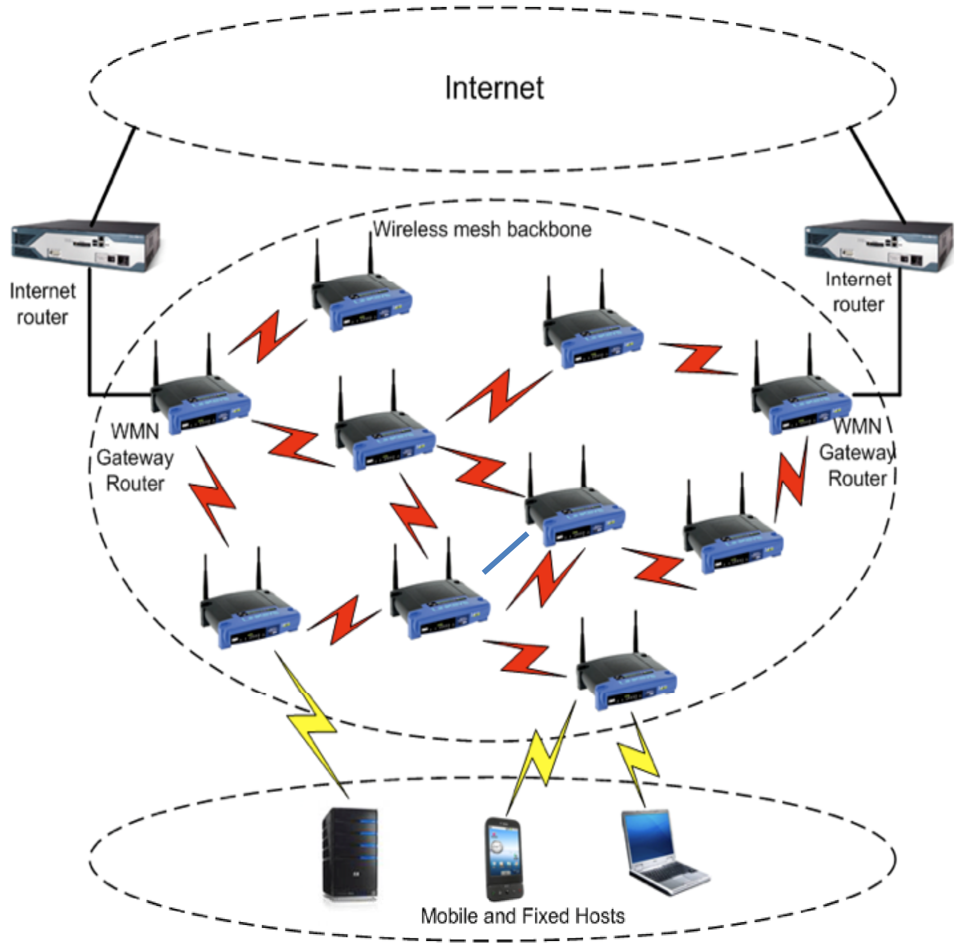
Outline

- Scenario
- Opportunistic Routing and Intra-session NC
- Optimization Model
  - Multiple sources, lossless links
  - Multiple sources, lossy links
- Decomposition and Interpretation
- Conclusion

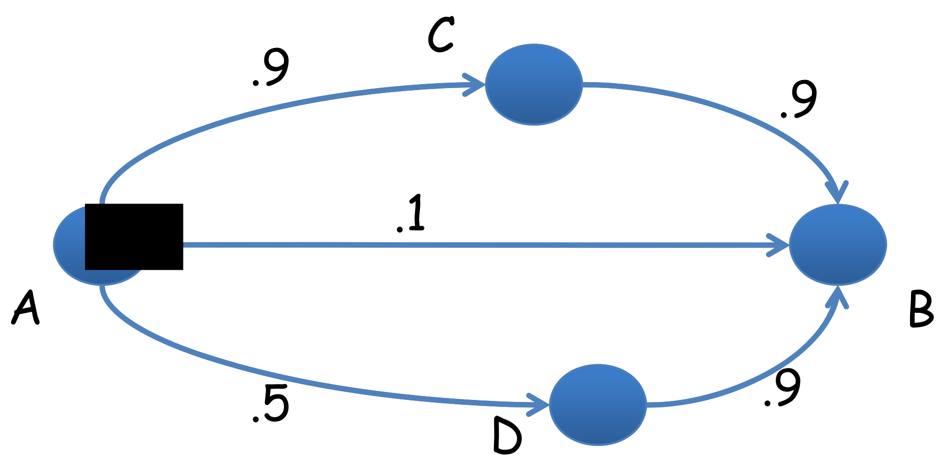
Wireless Mesh Networks

- Focus on WMNs:
  - Multiple paths
  - Braodcast channel
  - Spatial reuse
  - Lossy links
  - MAC contention and interference



Opportunistic Routing

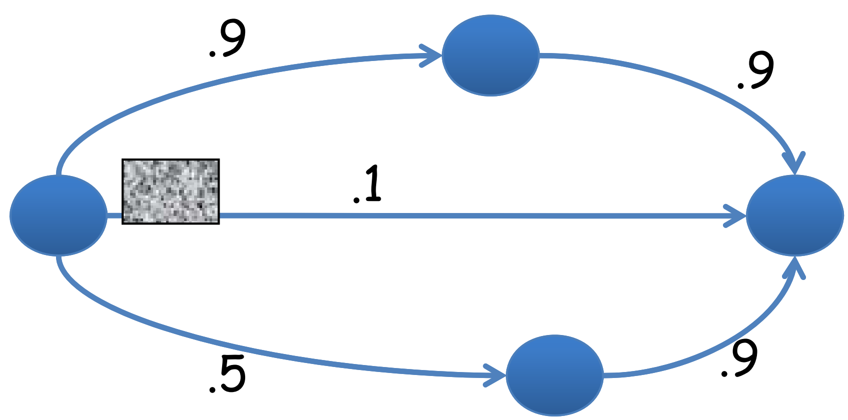
- Opportunistic Routing vs. Predetermined Routing
  - Next-hop node not chosen a priori
  - At each transmission a set of candidate nodes is selected
  - After packet transmission, candidate nodes (implicitly) coordinate to elect a forwarder



Biswas, Morris, "Opportunistic Routing in Multi-Hop Wireless Networks", SIGCOMM 2005

Opportunistic Routing and NC

- OR requires: signaling + coordinating candidate nodes
- Use intra-session NC to simplify the scheduling



Chachulski et al, "Trading Structure for Randomness in Wireless Opportunistic Routing", SIGCOMM 2007

Opportunistic Routing and NC

- OR+NC: new questions
  - When should each forwarder stop sending packets?
  - How about the source ?

- OR+NC: optimization models
  - [Radunovic et al. 2009] propose a primal-dual algorithm
    - Use hyper-graph
    - Requires introducing credit variables to separate flow control and routing variables
    - Use Lyapunov function to prove stability

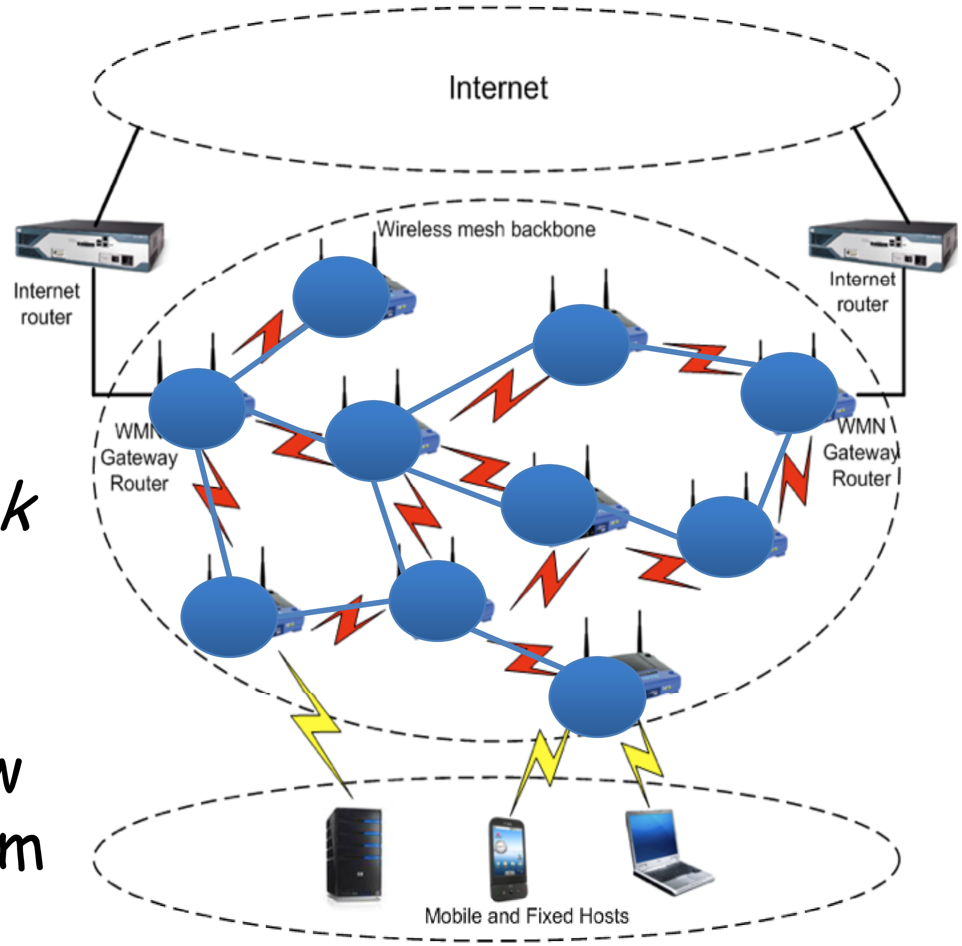
Optimization Model

Goal:

- Use node-specific variables to understand the interaction OR+NC with
  - Multiple sources, lossless links
  - Multiple sources, lossy links

Notation

- Model WMN as Graph:
  - |V|=N nodes, |E| edges
  - C, link capacity
  - K source-destination nodes: (s^k, d^k)
  - z_i^k : # of pkts of flow *k* sent by node *i*
  - $z_i = \sum_{k \in K} z_i^k$
  - $\sum_{j>i} z_j^k$: # pkts of flow *k* received by node *i* from "upstream" nodes



Optimization Model

Multiple sources, lossless links

$$\max_{z_i^k \geq 0, c_{ij} \geq 0} \sum_{k \in K} U(z_{s^k}^k)$$

s.t.

$$\sum_{j > d^k} z_j^k = z_{s^k}^k \qquad \forall k \in K$$

$$\sum_{j > i} z_j^k \geq z_i^k \qquad \forall i \neq s^k, k \in K$$

$$\sum_{k \in K} z_i^k \leq c_{ij} \qquad \forall i, \forall j \; : \; j < i$$

$$c \in \Pi$$

Decomposition
Lossless links

- Consider the (partial) dual: $\min_{p\geq 0, q} D(p,q)$
- Where:

$$\mathrm{D}(p,q) =$$

$$\max_{z_i^k \geq 0, c_{ij} \geq 0} \sum_{k\in K} U(z_{s^k}^k) - \sum_i \sum_{j<i} p_{ij} \Big(\sum_{k\in K} z_i^k - c_{ij} \Big) + $$

$$- \sum_{k\in K} q_{s^k} \Big(z_{s^k}^k - \sum_{j>d^k} z_j^k \Big)$$

$$\text{s.t.} \quad \sum_{j>i} z_j^k \geq z_i^k \qquad \forall i \neq s^k, k \in K$$

$$c \in \Pi$$

Decomposition

Lossless links

- D1:

$$\max_{z_{s^k}} \sum_{k\in K} \left(U(z_{s^k}^k) - (p_s + q_{s^k}) z_{s^k}\right)$$

Congestion Control

- D2:

$$\max_{z_i \geq 0, i\neq s} \sum_{k\in K} \left(q_{s^k} \sum_{j>d^k} z_j^k - \sum_{i\neq s}\sum_{j<i} p_{ij} z_i^k\right)$$

$$\text{s.t.} \quad \sum_{j>i} z_j^k \geq z_i^k \qquad \forall i \neq s$$

Routing

- D3

$$\max_{c\geq 0} \sum_{j<i} p_{ij} c_{ij}$$

$$\text{s.t.} \quad c \in \Pi$$

Wireless Interference

Decomposition

Lossless links

- D1: $$\max_{z_{s^k}} \sum_{k \in K} \left(U(z^k_{s^k}) - (p_s + q_{s^k}) z_{s^k}\right)$$

Congestion Control

- Admits a unique solution:

$$z^k_{s^k} = U'^{-1}(p_{s^k} + q_{s^k})$$

$$\sum_{j > d^k} z^k_j = z^k_{s^k}$$

Decomposition

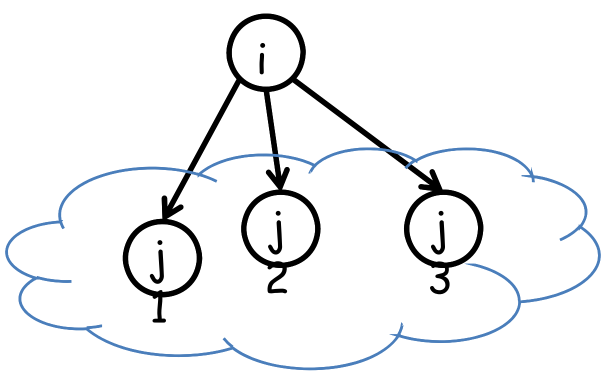
Lossless links

- D2:

$$\max_{z_i \geq 0, i \neq s} \sum_{k \in K} \Bigg(\sum_{\substack{i \neq s \\ i \notin \{j > d^k\}}} z_i^k \left(q^k_{out(i)} - q_i + p_i\right) + \sum_{\substack{i \neq s \\ i \in \{j > d^k\}}} z_i^k \left(q_s + q^k_{out(i)} - q_i - p_i\right)$$

$i \neq s$

$$q_{out(i)} = \sum_{j<i} q_j$$



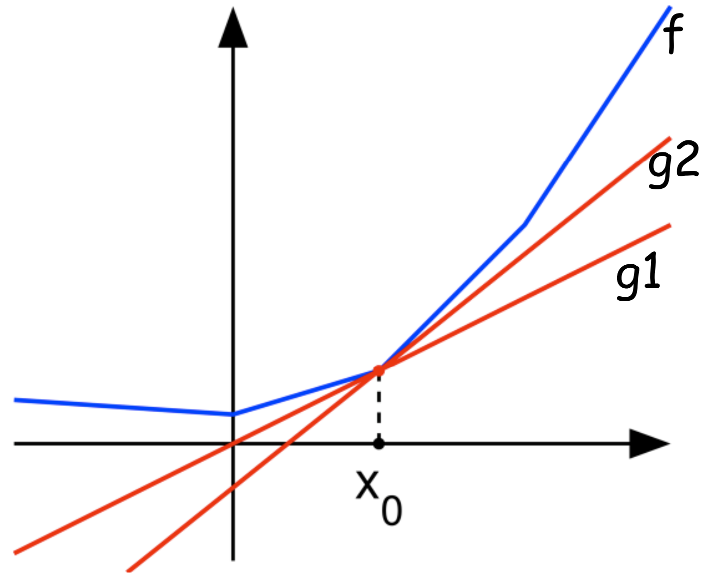
Feedback provided by
the *union* of
downstream nodes

Solving the Dual Problem

- Dual problem: $\min_{p \geq 0, q} D(p,q)$

can be solved using a sub-gradient method:

$$x^{(t+1)} = x^{(t)} - \alpha_t g(x^{(t)})$$



- Guaranteed to converge

(provided the primal problem is convex)

Solving Dual Problem

- Primal problem:

$$\max_{z_i^k \geq 0, c_{ij} \geq 0} \sum_{k \in K} U(z_{s^k}^k)$$

$$\text{s.t.} \quad \sum_{j > d^k} z_j^k = z_{s^k}^k \qquad \forall k \in K$$

$$\sum_{j > i} z_j^k \geq z_i^k \qquad \forall i \neq s^k, k \in K$$

$$\sum_{k \in K} z_i^k \leq c_{ij} \qquad \forall i, \forall j \;:\; j < i$$

$$c \in \Pi$$

- Update rule:

$$p_{ij}(t+1) = \left[p_{ij}(t) + \alpha_{ij}(z_i(t) - c_{ij}(t))\right]^+$$

$$q_{s^k}(t+1) = q_{s^k}(t) + \beta_{s^k}\left(z_{s^k}^k(t) - \sum_{j > d^k} z_j(t)\right)$$

Solving Dual Problem

- Source rate decreases:

$$\max_{z_{s^k}} \sum_{k \in K} \left(U(z^k_{s^k}) - (p_s + q_{s^k}) z_{s^k} \right)$$

- Destination's neighbors increase their rate
- This propagates backward to all active nodes

$$\max_{z_i \geq 0, i \neq s} \sum_{k \in K} \left(q_{s^k} \sum_{j > d^k} z^k_j - \sum_{i \neq s} \sum_{j < i} p_{ij} z^k_i \right)$$

$$\text{s.t.} \quad \sum_{j > i} z^k_j \geq z^k_i \qquad \forall i \neq s$$

Optimization Model

Multiple sources, lossy case

- Notation:
  - σ_{ij} : loss probability between *i* and *j*
  - $z_i^k(1 - \prod_{j<i} \sigma_{ij})$: (avg) number of packets received by any of i's neighbors

Optimization Model

Multiple sources, lossy links

$$\max_{z_i^k \geq 0, c_{ij} \geq 0} \sum_{k \in K} U(z_s^k)$$

$$\text{s.t.} \quad \sum_{j>d} z_j^k \bar{\sigma}_{jd} = z_s^k \Big(1 - \prod_{j<s} \sigma_{sj}\Big) \qquad \forall k$$

$$\sum_{j>i} z_j^k \bar{\sigma}_{ji} \geq z_i^k \Big(1 - \prod_{j<i} \sigma_{ij}\Big) \qquad \forall k, i \neq s$$

$$\sum_{k \in K} z_i^k \bar{\sigma}_{ij} \leq c_{ij} \qquad \forall i, \forall j : j < i$$

$$c \in \Pi$$

Optimization Model

Dual Problem

$$D(p,q) = \max_{z\geq 0, c\geq 0} \sum_{k\in K} \Bigg[U(z_s^k) - \sum_i \sum_{j<i} p_{ij}\Big(z_i^k \bar{\sigma}_{ij} - c_{ij}\Big) + \\ - \sum_{k\in K} q_{s^k}\Big(\sum_{j>d} z_j^k \bar{\sigma}_{jd} - z_s^k (1 - \prod_{j<s} \sigma_{sj})\Big)\Bigg]$$

$$\sum_{j>i} z_j^k \bar{\sigma}_{ji} \geq z_i^k (1 - \prod_{j<i} \sigma_{ij}) \qquad \forall i \neq s$$

$$c \in \Pi$$

Conclusion

- Present a simple model for OR+NC using node-specific variables:
  - we do not need to model predetermined routes.
- It allows to derive how:
  - E2E: the source rate adapts to end-to-end feedback
  - H2H: each intermediate node coordinates with the union of its downstream neighbors
- Future work:
  - Further analysis on OR+NC interactions
  - Protocol design

Thank you!