1. The autocorrelation function of a random process $X(t)$ is
\[ \phi_X(\tau) = \frac{1}{2} N_0 \delta(\tau). \]
Such a random process is called white noise. Suppose $X(t)$ is the input to an ideal bandpass filter with the frequency response
\[ |H(f)| = \begin{cases} 
1 & -f_c - B/2 \leq f \leq -f_c + B/2, \\
1 & f_c - B/2 \leq f \leq f_c + B/2, \\
0 & \text{elsewhere}.
\end{cases} \]
Determine the total noise power at the output of the filter.

2. Use the Chernoff bound to show that $Q(x) \leq e^{-x^2/2}$ where $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-t^2/2} dt, x > 0$.

3. Determine the mean, the autocorrelation sequence, and the power density spectrum of the output of a system with unit sample response
\[ h(n) = \begin{cases} 
1 & n = 0, \\
-2 & n = 1, \\
1 & n = 2, \\
0 & \text{otherwise}
\end{cases} \]
where the input $X(n)$ is a wide sense stationary random sequence with variance $\sigma_X^2$.

4. The autocorrelation sequence of a discrete-time random process is $\phi(k) = (\frac{1}{2})^{|k|}$. Determine its power density spectrum.

5. Show that the functions
\[ f_k(t) = \frac{\sin \left[ 2\pi W \left( t - \frac{k}{2W} \right) \right]}{2\pi W \left( t - \frac{k}{2W} \right)} \quad k = 0, \pm 1, \pm 2, \ldots \]
are orthogonal over the interval $(-\infty, \infty)$, i.e.,
\[ \int_{-\infty}^{\infty} f_k(t) f_j(t) dt = \frac{1}{2W} \delta_{kj}. \]
Therefore, the sampling theorem construction formula may be viewed as a series expansion of the bandlimited signal $s(t)$, where the weights are samples of $s(t)$ and the the $\{f_k(t)\}$ are the set of orthogonal functions used in the series expansion.