

A Linear-Complexity Distortion Optimal Scheme for the Transmission of Packetized Progressive Bitstreams

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Abstract—We propose a novel distortion minimization technique for the transmission of a packetized progressive bitstream. The optimality of our proposed algorithm is analytically proved for a class of sources satisfying a stated condition. It is shown that Gauss-Markov sources belong to the latter class for which the algorithm is optimal. We show that our proposed optimization technique is robust and has a linear complexity in the transmission rate. Simulation results show the effectiveness of our proposed algorithm.

Index Terms—Progressive Coding, Joint Source-Channel Coding, Rate Allocation.

I. INTRODUCTION

Progressive transmission with embedded coding has been proven to be a viable framework for delivering multimedia content over noisy channels. In essence, embeddedness is the ability of a source coder to allow a decoder to progressively reconstruct its data at different bit rates from the prefixes of a single bitstream. This capability, however, comes at the expense of high sensitivity to transmission noise and the possibility of error propagation. Therefore, progressive transmission over noisy channels has to be accompanied with appropriate channel coding or joint source-channel coding schemes.

Joint source-channel coding of progressive bitstreams has been studied by many researchers. The problem usually manifests itself as an optimization task that attempts to either minimize the overall distortion, or maximize the source coding rate. While the former problem is commonly referred to as a distortion-optimal problem, the latter is known as a rate-optimal problem. Concatenated coding [2], [6], dynamic programming [2], [3], [4], exhaustive search [6], and Gradient-based optimization [1], [10], [9] are among the techniques used to solve different variants of these optimization problems. In [10], we considered distortion-optimal transmission of progressive images over channels with bit errors and packet erasures.

Practical implementation of a multimedia transmission system requires low complexity, robustness and a near optimal performance. Gradient-based techniques [1], [10] are sensitive to the initial conditions, in the best case have super-linear ($\mathcal{O}(N \log N)$) complexity, and may fail to find the global optimum. The dynamic programming techniques of [2], [3], [4] find optimal or sub-optimal solutions but with a

quadratic complexity. Rate-optimal techniques [4], [8] offer low-complexity alternatives but are usually sub-optimal in the distortion sense. The low-complexity PLS algorithm of [7] solves a rate-optimal problem, performs a local search for lower-distortion solutions, and may converge to a local optimum.

In this letter, we propose a low-complexity distortion minimization technique that has no convergence issues. We analytically prove that for a class of sources satisfying a given condition, our proposed linear-complexity algorithm always converges to the globally optimal solution. As an example, Gauss-Markov sources satisfy the optimality condition of our algorithm. The structure of this letter is as follows. In Section II, we formulate our distortion-optimal problem and present our proposed solution to it. In this section, we also prove the optimality of our algorithm. We present our simulation results in Section III. Finally, Section IV concludes this letter.

II. ANALYSIS

A. Problem Description

We consider the problem of transmitting a progressively encoded bitstream using N packets of length L using a budget of $B_T = NL$ symbols. As shown in Fig. 1(a), the i^{th} packet consists of C_i channel coding symbols together with b_i data symbols. The expected distortion of the transmitted bitstream can be written as [10]:

$$J = \sum_{i=1}^{N+1} \Psi_i D_{i-1} \prod_{j=1}^{i-1} (1 - \Psi_j) \quad (1)$$

where D_i is the distortion associated with the first i packets, and Ψ_i is the failure probability of packet i . The boundary conditions for the above equations are $\Psi_{N+1} = 1$ and $D_0 = \sigma^2$, where σ^2 is the source variance. Variables D_i and Ψ_i depend on the choice of the source coder, channel error statistics, and the choice of C_i 's. The goal is to find a set of C_i 's that minimizes this distortion. A global search over all possible parity combinations requires L^N cost function evaluations which is impractical except for a very small L and N . We can reduce this number by making an Unequal Error Protection (UEP) assumption in the form of $C_1 \geq C_2 \geq \dots \geq C_N$ which is quite reasonable for a progressively encoded bitstream. Even with the UEP assumption, the number of function evaluations increases very quickly with L and N . As an alternate approach, one can try to solve a constrained

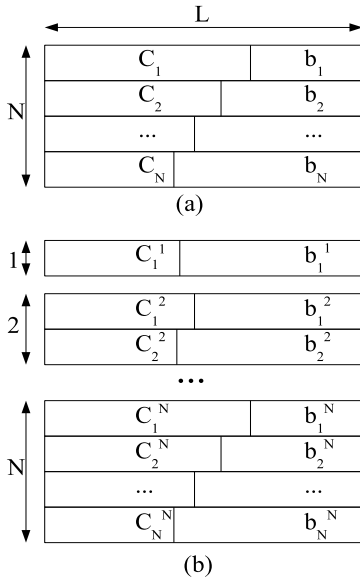


Fig. 1. (a) Original Optimization Problem (b) Packet by Packet Optimization

nonlinear optimization problem in the form of:

$$\min_{C_1, \dots, C_N} J \quad (2)$$

$$\text{Subject To: } 0 \leq C_i < L, \quad i \in \{1, \dots, N\} \quad (3)$$

In [10], we solve this nonlinear optimization problem using the Sequential Quadratic Programming (SQP) technique. The range of potential challenges involved with solving the above problem include sensitivity to the choice of initial conditions, high time-complexity, and convergence to non-global optimal points.

B. Distortion Minimization Algorithm

We propose a packet-by-packet distortion minimization algorithm as shown in Fig. 1(b). Instead of solving an optimization problem with N decision variables, we solve N single-variable optimization problems. Although the i^{th} optimization problem consists of i packets, only the first packet has to be optimized. Let us use superscript i as an index for the i^{th} optimization problem. Then, C_j^i and Ψ_j^i represent the number of channel coding symbols and the failure probability of the j^{th} packet in the i^{th} optimization problem, respectively.

Consider the first optimization problem with a single packet. We can simply find the optimal solution by searching over all L possible values $\{0, \dots, L-1\}$ for C_1^1 . Now, consider the second optimization with two packets. In general, optimizing C_2^2 depends on C_1^1 since the distortion D_2^2 associated with receiving two packets includes the effects of the number of source symbols of the first packet, namely $L - C_1^1$. However, we will show that the distortion optimization of the two packets can be decoupled from one another under some conditions. By doing so, we can use the result of the single-packet optimization and set $C_2^2 = C_1^1$. Generalizing this result to the i^{th} optimization problem, we have:

$$C_j^i = C_{j-1}^{i-1} \quad j = 2, \dots, i \quad (4)$$

We can now formalize our optimization algorithm as the following:

- Find $C_1^1 \in \{0, 1, \dots, L-1\}$ that minimizes the expected distortion for a single packet.
- For $i = 2, \dots, N$
 - Set $C_j^i = C_{j-1}^{i-1} \quad j = 2, \dots, i$.
 - Find $C_1^i \in \{0, \dots, L-1\}$ that minimizes the expected distortion for i packets.
- $\{C_1^N, \dots, C_N^N\}$ is the optimal solution.

In the next section, we will prove that our algorithm is optimal for the class of sources satisfying the following condition:

$$D(n + \Delta n) = \frac{D(n)D(\Delta n)}{D(0)} \quad \Delta n \leq L \quad (5)$$

In the above equation, $D(n)$ is the distortion associated with the first n information symbols. As an example, a Gauss-Markov source has a D-R function in the form of an exponential $D(n) = \alpha e^{-\beta n}$ that satisfies the optimality condition of our algorithm. Our simulation results show that our proposed algorithm also provides good results for sources characterized by piecewise exponential D-R functions. The latter condition is practically met for a wide range of sources and as a result, our algorithm can be applied to all such sources.

The worst-case complexity of this algorithm is $\mathcal{O}(NL)$. In other words, it is linear in both N and L . Notice that by invoking the UEP assumption, we can limit the search space for C_1^i to the set $\{C_2^i, \dots, L-1\}$. This will usually not compromise the optimality of the algorithm. The search space can be further limited based on the allowable code rates.

In deriving the above algorithm, we have made no assumption on the channel and channel coding scheme used. Our proposed algorithm is applicable to any system so long as the bitstream is progressively encoded. While the choice of the source coder, channel coder, and the transmission medium will be reflected in the variables D_i and Ψ_i , it does not change the proposed algorithm.

The by-product of the recursive optimal solution for N packets is the optimal solution for k packets where $1 \leq k \leq N-1$. This property is useful in applications involving different levels of Quality of Service (QoS) associated with different transmission rates. Performing the optimization for the maximum transmission rate automatically provides the optimal channel coding rates for all of the intermediate transmission rates.

Finally, we notice that the convergence of the search algorithm is guaranteed for all values of N and L . Consequently, the convergence of our algorithm is not affected by the choice of initial conditions unlike gradient-based algorithms.

C. Proof of Optimality

We prove the optimality of our algorithm using mathematical induction on the number of packets N . For $N = 1$, the number of parity symbols associated with the global optimum solution is found by conducting a search over all possibilities. Now, let us assume that we know how to find the optimal solution for N packets and we want to show that the optimal

solution for the last N packets in the $N + 1$ -packet problem is the same as our solution for the N -packet problem.

Since $L - C_{i-1} \leq L$, we can apply (5) to D_{i-1} :

$$D_{i-1} = D \left[\sum_{k=1}^{i-1} (L - C_k) \right] \quad (6)$$

$$= \frac{1}{D_0} D \left[\sum_{k=1}^{i-2} (L - C_k) \right] D(L - C_{i-1}) \quad (7)$$

By recursively applying (5) to (7) we get:

$$D_{i-1} = \frac{1}{D_0^{i-2}} \prod_{k=1}^{i-1} D(L - C_k) \quad (8)$$

Substituting (8) in (1), we can write the cost function of the N -packet problem as:

$$J^N = \sum_{i=1}^{N+1} \frac{\Psi_i^N}{D_0^{i-2}} \prod_{k=1}^{i-1} D(L - C_k^N) \prod_{j=1}^{i-1} (1 - \Psi_j^N) \quad (9)$$

By writing a similar expression for the cost function of the $N+1$ -packet problem, expanding the $i = 1$ term, and factoring out the $D(L - C_1^{N+1}) = D_1^{N+1}$ and $(1 - \Psi_1^{N+1})$ terms, we derive:

$$J^{N+1} = D_0 \Psi_1^{N+1} + (1 - \Psi_1^{N+1}) \frac{D_1^{N+1}}{D_0} J_e \quad (10)$$

where

$$J_e = \sum_{i=2}^{N+2} \frac{\Psi_i^{N+1}}{D_0^{i-3}} \prod_{k=2}^{i-1} D(L - C_k^{N+1}) \prod_{j=2}^{i-1} (1 - \Psi_j^{N+1}) \quad (11)$$

J_e is a partial cost function that is independent of C_1^{N+1} and only depends on the N variables $\{C_2^{N+1}, \dots, C_{N+1}^{N+1}\}$. Since J^{N+1} is a non-decreasing function of J_e , i.e., $\frac{\partial J^{N+1}}{\partial J_e} =$

$(1 - \Psi_1^{N+1}) \frac{D_1^{N+1}}{D_0} > 0$, the optimal solution for the latter variables is obtained by solving the N -packet minimization problem with the cost function J_e . Let $\Psi'_i = \Psi_{i+1}^{N+1}$ and $C'_k = C_{k+1}^{N+1}$. By replacing Ψ'_i and C'_k in (11) and re-indexing the variables i, j, k to start from 1, we will arrive at:

$$J_e = \sum_{i=1}^{N+1} \frac{\Psi'_i}{D_0^{i-2}} \prod_{k=1}^{i-1} D(L - C'_k) \prod_{j=1}^{i-1} (1 - \Psi'_j) \quad (12)$$

By comparing (9) and (12), we note that except for the variable naming, J_e and J^N represent the same cost function with the same set of constraints. The two optimization problems have the same solution, and as a result:

$$C_{j_{opt}}^{N+1} = C'_{j-1_{opt}} = C_{j-1_{opt}}^N \quad j = 2, \dots, N+1 \quad (13)$$

III. NUMERICAL RESULTS

In this section, we present the simulation results of our proposed algorithm. We use the 512×512 gray scale Lena image encoded progressively using the SPIHT encoder [5]. The operational D-R curve of the source coder is implemented as a lookup table whose entries can be obtained during the encoding process [5]. We have utilized the Gilbert-Elliot error model to represent the time-varying bit error rate of a fading

channel. The transition probabilities for the GOOD to BAD and BAD to GOOD state transitions are 0.00127 and 0.125, respectively. Per state symbol error rates are derived from the per state signal-to-noise ratios. We have used the relationship $SNR_G = 10SNR_B$ in order to distinguish between the signal-to-noise ratios of the GOOD and BAD states. Channel coding is done using rate compatible punctured Reed-Solomon codes over $GF(256)$ with a symbol size of one byte. We refer the reader to [10] for a detailed description of the channel model and the calculation of the loss probabilities Ψ_i .

For $L = 50$, $SNR_G = 7$ dB and $N \leq 10$ packets, we have compared the output of our algorithm with an exhaustive search over all possibilities. Fig. 2(a) shows the operational D-R curve of the source coder with resolutions $\Delta R = 1$ and $\Delta R = 50$ bytes. Resolution parameter ΔR represents the accuracy of the available D-R curve estimate. We used $\Delta R = 50$ for the optimization and $\Delta R = 1$ for the exhaustive search. Fig. 2(b) shows that except for the extremely low rate regime, our proposed algorithm is capable of finding the global optimum. This example also shows that our proposed algorithm provides good results when only a coarse estimate of the D-R curve is available.

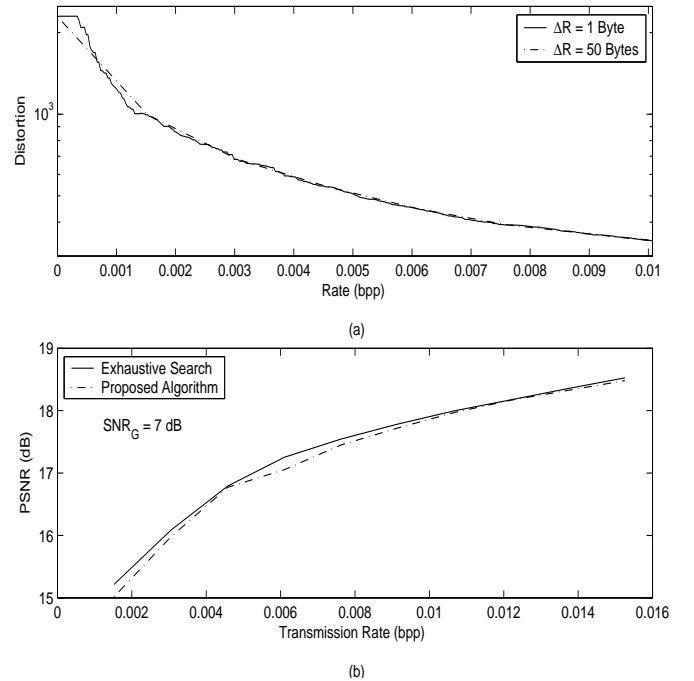


Fig. 2. (a) Operational D-R curve of the source coder (b) Exhaustive search and the proposed algorithm results

We used a packet length of $L = 100$ in the rest of the examples. Fig. 3 compares the results of our proposed algorithm with those of Sequential Quadratic Programming (SQP) technique used in [10]. The SQP algorithm shows convergence issues at $SNR_G = 9$ dB, which is not present in our proposed algorithm.

Finally, Fig. 4 compares the expected PSNR based on our proposed optimization algorithm with the PSNR of the simulated transmitted system. Every point in this curve is an average value taken over 200 simulations. The SPIHT source

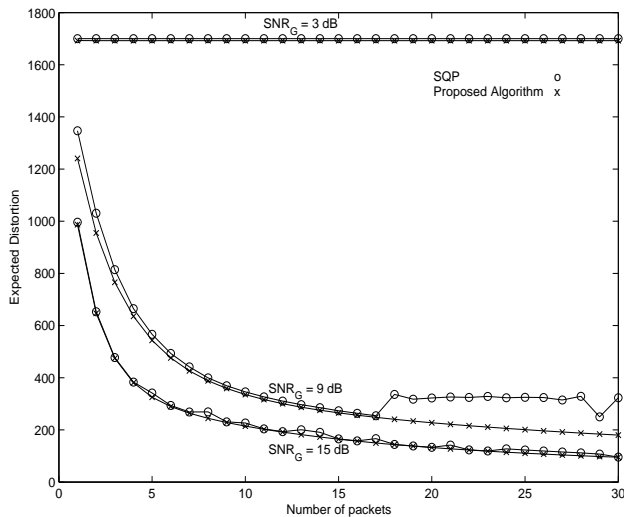


Fig. 3. Distortion Minimization Results

coder performance is also shown as a point of reference. The close agreement of the above curves validates our overall simulation model.

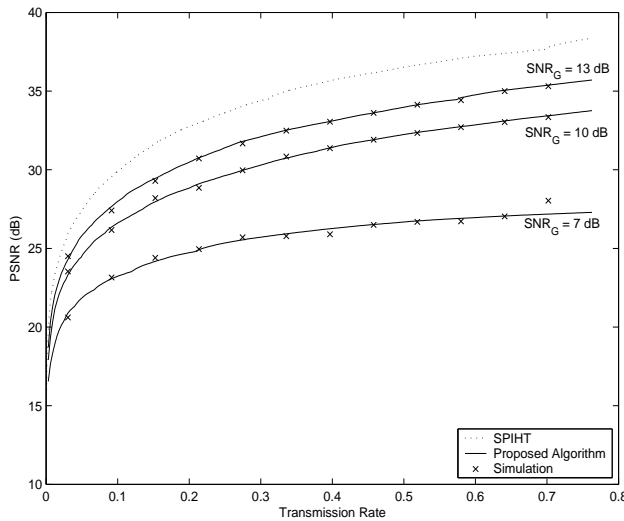


Fig. 4. Simulation Results

IV. CONCLUSIONS

In this letter, we proposed a novel distortion minimization technique for the transmission of a packetized progressive bitstream. Our proposed technique is applicable to any transmission system so long as the bitstream is progressively encoded and the packet failure rates can be calculated. We proved the optimality of our proposed technique for the class of sources, including Gauss-Markov sources, that satisfy (5). We numerically showed the effectiveness of our proposed algorithm for arbitrary progressively encoded sources. We showed that our proposed optimization technique has a linear complexity in the transmission rate and unlike many previously known techniques, is free of numerical problems associated with the convergence or sensitivity to the choice

of the initial conditions. We showed the advantages of our proposed technique through simulations.

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