UCI EECS
EECS141B
Spring 2008

HOMEWORK 2
(Due 5/1/2008)

1. In a quaternary communication system, one of the four signals

$$\begin{aligned} s_1(t) &= 0, \\ s_2(t) &= A_c \cos(2\pi f_c t), \\ s_3(t) &= A_c \sin(2\pi f_c t), \\ s_4(t) &= A_c \cos(2\pi f_c t) + A_c \sin(2\pi f_c t) \end{aligned}$$

   is transmitted during the interval $0 \le t \le T$, $T = n/f_c$ where n is a positive integer. All signals are equal to 0 outside $0 \le t \le T$. The transmitted signal goes through an additive white Gaussian noise channel with zero mean and power spectral density $N_0/2$.

   (a) Draw the signal constellation.

   (b) Calculate the nearest neighbor bound for the probability of symbol error incurred by the optimum receiver.

2. In a quaternary communication system, one of the four signals

$$s_i(t) = A_i \cos(2\pi f_c t) + A_i \sin(2\pi f_c t)$$

   is transmitted during $0 \le t \le T$, $T = n/f_c$ where n is a positive integer, and A_i is $-3, -1, 1, 3$ for i equal to $1, 2, 3, 4$, respectively. The transmitted signal goes through an additive white Gaussian noise channel with zero mean and power spectral density $N_0/2$.

   (a) Consider the orthonormal basis signal set

$$\phi_1(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_c t) \qquad \phi_2(t) = \sqrt{\frac{2}{T}} \sin(2\pi f_c t)$$

   for $0 \le t \le T$. All signals are equal to 0 outside $0 \le t \le T$. Using this set of basis functions, draw the optimum receiver, draw the signal constellation, show the optimum decision regions, and calculate the probability of error associated with the optimum receiver.

   (b) Can you simplify the receiver in part (a) above? If yes, draw the new optimum receiver, draw the signal constellation, show the optimum decision regions, and calculate the probability of error associated with the optimum receiver.

3. During $0 \leq t \leq T$, one of the 64 signals

$$s_i(t) = \sum_{k=0}^{2} s_{ik} \cos(2\pi(f_c + k\Delta f)t)$$

where $T = n/f_c$, n is a positive integer, $\Delta f = 1/2T$, $s_{ik} \in \{-3, -1, 1, 3\}$, and $i = 1, 2, \ldots, 64$ is transmitted on an additive white Gaussian signal with zero mean and power spectral density $N_0/2$. There is a one-to-one correspondence between a given i and the triplet s_{i0}, s_{i1}, s_{i2}.

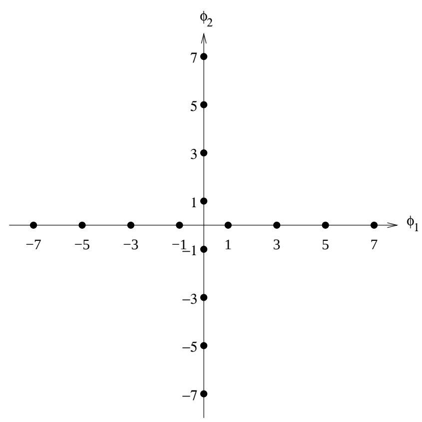
   (a) Calculate the exact probability of error.

   (b) Calculate the nearest neighbor bound for the probability of symbol error.

   (c) Compare your responses in parts (a) and (b), and comment if the nearest neighbor bound is a good approximation of the exact probability of error.

4. Let P_{eI} and P_{eQ} denote the probabilities of symbol error for the in-phase and quadrature channels of a narrowband digital communication system. Calculate the probability of symbol error for this system in terms of P_{eI} and P_{eQ}.

5. For the signal constellation shown in the figure below, determine the optimum decision boundaries for the detector, assuming that the signal-to-noise ratio is sufficiently high so that errors only occur between adjacent points.



6. Suppose that binary PSK is used for transmitting information over an additive white Gaussian noise channel with zero mean and power spectral density $\frac{1}{2}N_0 = 10^{-10}$ W/Hz. The transmitted signal energy is $E = \frac{1}{2}A^2T$ where T is the bit interval and A is the signal amplitude. Determine the signal amplitude required to achieve a bit error probability of 10^{-6} when the data rate is

   (a) 10 kb/s,

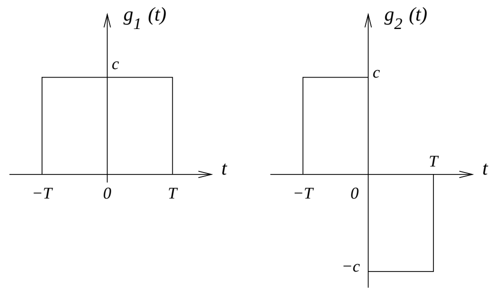
   (b) 100 kb/s,

   (c) 1 Mb/s.

7. Consider the signals $g_1(t)$ and $g_2(t)$ shown below.

   (a) Calculate the power spectra $|G_1(f)|^2$ and $|G_2(f)|^2$.

   (b) Which one would you prefer to use as a pulse shaping function $g(t)$ in a digital modulation system? Why?



8. Consider the channel frequency response $X(f)$ in the figure below.

   (a) Does this channel satisfy the Nyquist condition? Why?

   (b) What is the impulse response $x(t)$ for this channel?

   (c) Show by direct evaluation of the impulse response that $x(kT) = \delta_k$.

   (d) Compare this $x(t)$ with that of raised cosine. Which one would you prefer? Why?