

The topic of Fractionally Spaced Equalization was introduced in the following papers

- [1] G. Ungerboeck, "Fractional Tap-Spacing Equalizer and Consequences for Clock Recovery in Data Modems," IEEE Trans. Comm., Vol. 24, pp. 856-864, August 1976.
- [2] S. U. H. Qureshi and G. D. Forney, "Performance Properties of a  $T/2$  Equalizer," Nat. Telecom. Conf. Record, pp. 11.1.1-11.1.14, Los Angeles, CA, December 1977,
- [3] R. D. Gitlin and S. B. Weinstein, "Fractionally-Spaced Equalization: An Improved Digital Transversal Equalizer," Bell System Tech. Journal, Vol. 60, pp. 275-296, February 1979.

In the book Digital Communication, Lee & Messerschmitt discuss the formula for  $Y_T(f)$  with a sampling delay  $\tau_0$  for excess bandwidth less than 100%:

$$Y_T(f) = \frac{1}{T} e^{j2\pi f \tau_0} \left[ |H(f)|^2 + |H(f - \frac{1}{T})|^2 e^{-j2\pi \frac{\tau_0}{T}} \right]$$

$$= \frac{1}{T} e^{j2\pi f \tau_0} |H(f)|^2 \left[ 1 + \alpha e^{-j2\pi \frac{\tau_0}{T}} \right] \quad 0 \leq f \leq 1/2T$$

where  $\alpha = \frac{|H(f - \frac{1}{T})|^2}{|H(f)|^2}$ , which is usually less than 1.

The magnitude of the complex number in brackets above is

$$\left| 1 + \alpha e^{-j2\pi \frac{\tau_0}{T}} \right| = \sqrt{1 + \alpha^2 + 2\alpha \cos\left(2\pi \frac{\tau_0}{T}\right)}$$

whose minimum is  $1 - \alpha$  when  $\tau_0 = T/2$ . When  $\tau_0 = T/2$  and for  $f$  close to  $\frac{1}{2T}$ , there will be a deep null in  $Y_T(f)$ . For other values of  $\tau_0$ , the spectrum will exhibit a null, which

will cause noise enhancement.

There is another problem that the fractionally spaced equalizer solves. It inherently has more taps than the symbol rate equalizer. This means it has more degrees of freedom to equalize a channel. Folded channel spectra can have rapid transitions near the band edges that are difficult to equalize. The extra degrees of freedom that the fractionally spaced equalizer offer help in equalizing such spectra with rapid transitions.

The following paper is recommended reading for Chapters 1-5 of the notes:

S. U. H. Qureshi, "Adaptive Equalization," Proceedings of the IEEE, Vol. 73, pp. 1349-1387, Sept. 1985.