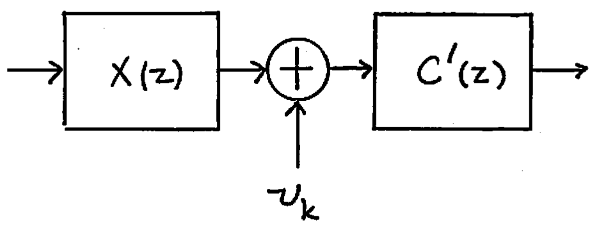
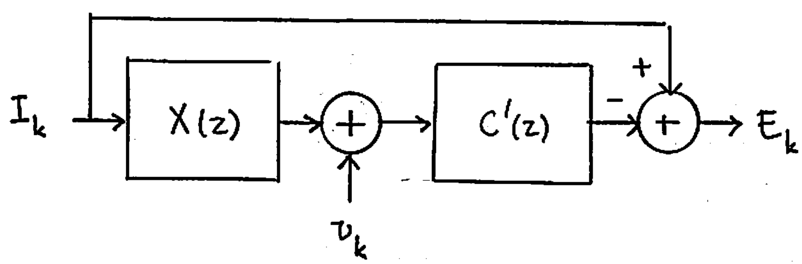
Infinite Length Mean Square Error DFE

We will first discuss an alternative derivation of LE-MSE that will be useful in deriving the DFE-MSE. Recall the model for LE

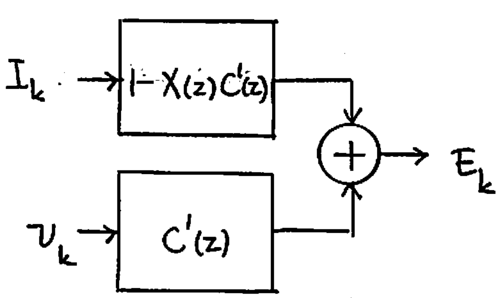


PSD $N_0 X(z)$

We wish to minimize the power in error E_k in the model below.



which is equivalent to



Let's calculate E_k in $e^{j\omega T}$ domain, call the transform $S_E(e^{j\omega T})$ (because $E(e^{j\omega T})$ can be confusing). Note $E[I_k^2]=1$, $E[I_k]=0$.

$$S_E(e^{j\omega T}) = |1 - X(e^{j\omega T})C'(e^{j\omega T})|^2 + N_0 X(e^{j\omega T}) |C'(e^{j\omega T})|^2$$

Note $X(e^{j\omega T})$ is real. By suppressing $e^{j\omega T}$, we can write

$$
\begin{aligned}
S_E &= (1 - XC')(1 - XC'^*) + N_o X |C'|^2 \\
&= 1 - X(C' + C'^*) + X^2 |C'|^2 + N_o X |C'|^2 \\
&= 1 - X(C' + C'^*) + (X + N_o) X |C'|^2 \\
&= 1 + X(X + N_o)\left[|C'|^2 - \frac{C' + C'^*}{X + N_o} \right] \\
&= 1 + X(X + N_o)\left[|C'|^2 - \frac{C' + C'^*}{X + N_o} + \frac{1}{(X + N_o)^2} \right] - \frac{X}{X + N_o} \\
&= X(X + N_o)\left| C' - \frac{1}{X + N_o} \right|^2 + \frac{N_o}{X + N_o}
\end{aligned}
$$

The first term is nonnegative and can be forced to zero by choosing

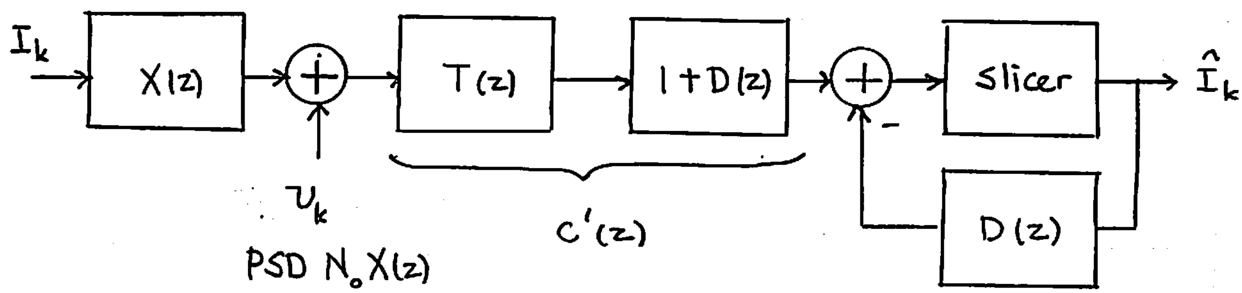
$$C'(z) = \frac{1}{X(z) + N_o}$$

and then the minimum MSE becomes

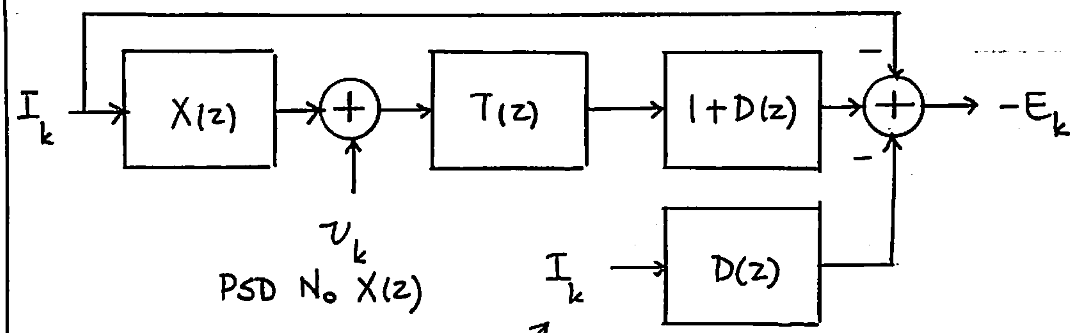
$$J_{min} = \frac{T N_o}{2\pi} \int_{-\pi/T}^{\pi/T} \frac{N_o}{X(e^{j\omega T}) + N_o} \, d\omega$$

as discussed previously.

We will now use this result to derive the DFE-MSE. The design is similar to that of DFE-ZF. We employ a filter $T(z)$ where we employed $1/X(z)$ in DFE-ZF. But in this case, we will want to design $T(z)$.

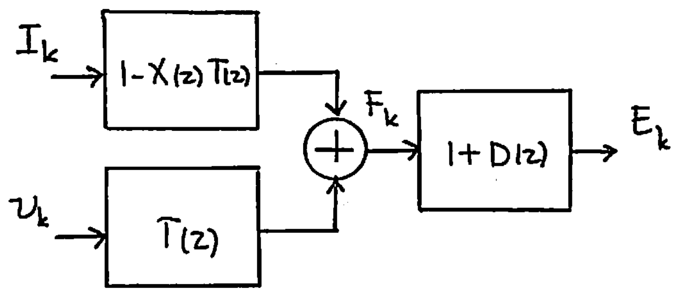


We wish to minimize the power in E_k (or $-E_k$) below.



(Assume $\hat{I}_k = I_k$, which will be in correct operation.)

This block diagram is equivalent to (show)



We wish to minimize the power in E_k. We can actually do this in two steps: Consider E_k and F_k. We can move to the $e^{j\omega T}$ domain analysis and carry out the minimization for each ω, as done for the LE-MSE. Then, the minimization can be carried out in two steps:

1. Optimize $T(e^{j\omega T})$ to minimize $S_F(e^{j\omega T})$, and then
2. Optimize $D(z)$ to minimize the power in E_k.

But, we know the solutions to both of these problems. Optimization of $T(e^{j\omega T})$ to minimize $S_F(e^{j\omega T})$ is exactly the same as the optimization of $C'(e^{j\omega T})$ to minimize the signal at the output of the adder in our alternate derivation of LE-MSE

As a result,

$$T(z) = \frac{1}{X(z) + N_0} .$$

The resulting power spectral density of F_k becomes

$$\frac{N_0}{X(z) + N_0} .$$

The second problem is a spectral factorization which states $D(z)$ should be chosen such that

$$\frac{N_0}{X(z) + N_0} \left(1 + D(z)\right)\left(1 + D^*(1/z^*)\right) = J_{min} \quad (\text{DFE-MSE})$$

The solution technique is the same as DFE-ZF, except one replaces $X(z)$ with $X(z) + N_0$.