UCI EECS
EECS141B
Spring 2009

HOMEWORK 4
(Will not be collected)

1. An analog signal with 4 kHz bandwidth is sampled at 1.25 times the Nyquist rate, and each sample is quantized into one of 256 equally likely levels. The successive samples are statistically independent.
   (a) After sampling and quantization, how many bits per second does the source generate?
   (b) Can the output of this source be transmitted without error over an AWGN channel with a bandwidth of 10 kHz and an SNR of 20 dB?
   (c) Find the SNR required for error-free transmission for part (b).
   (d) Find the bandwidth required for an AWGN channel for error-free transmission of the output of this source if the SNR ratio is 20 dB.

2. Consider the (8, 4) linear systematic code with the generator matrix

$$\mathbf{G} = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

   (a) List all the codewords generated by this code together with their Hamming weight.
   (b) How many bit errors can this code correct?
   (c) Calculate the parity-check matrix for this code.
   (d) Calculate the syndrome values associated with all bit errors in your response to (b) above.
   (e) Assume **r** = (0 1 0 1 1 1 0 1) is received. Calculate the syndrome.
   (f) What is the corresponding error pattern?
   (g) What is the corrected codeword?
   (h) What is the transmitted message?

3. In a *repetition code* every binary symbol is repeated n times, where $n = 2m + 1$ is an odd integer. There are only two codewords in the repetition code, an all-0 codeword and an all-1 codeword. Consider a repetition code with $n = 5$.
   (a) Construct the generator matrix **G** for this (5, 1) block code.
   (b) Using **G** find all all codewords and their Hamming weights.
   (c) How many bit errors can this code correct?

(d) Find the parity-check matrix **H** for this code.

(e) Calculate the syndrome values associated with all bit errors in your response in (c) above.

(f) Assume **r** = (1 0 1 0 0) is received. What is the syndrome, corresponding error pattern, corrected codeword, and transmitted message?

(g) The code has a special structure due to repetition. Can you propose a decoding technique simpler than syndrome decoding, employing this special structure?

(h) Assume that every bit transmitted to the channel has a probability of being in error equal to p. Show that the probability of the message being in error is equal to

$$P_e = \sum_{i=3}^{5} \binom{5}{i} p^i (1-p)^{5-i}.$$

(i) Let $p = 0.01$, calculate P_e.

4. The polynomial $X^7 + 1$ can be factored as

$$X^7 + 1 = (1 + X)(1 + X^2 + X^3)(1 + X + X^3).$$

We will design a (7,3) code based on the generator polynomial

$$g(X) = (1 + X)(1 + X^2 + X^3).$$

(a) Calculate a nonsystematic generator matrix based on $g(X)$.

(b) Convert the nonsystematic generator matrix of part (a) into a systematic generator matrix **G**.

(c) Calculate all codewords generated by **G** and their Hamming weights.

(d) How many bit errors can this code correct?

(e) Calculate the parity-check matrix **H** corresponding to **G**.

(f) Calculate the syndrome values associated with all bit errors in your response to (d) above.

(g) Draw the encoder circuit associated with $g(X)$.

(h) Assume the message **m** = (1 0 1) is fed into the encoder circuit and calculate its output.

(i) Draw the syndrome circuit associated with $g(X)$.

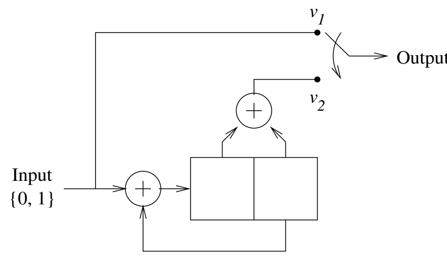
(j) Assume **r** = (0 0 1 1 0 0 1) is fed into the syndrome circuit and calculate its output.

(k) Based on this syndrome, what is the error pattern?

(l) Calculate the corrected codeword.

(m) What is the transmitted message **m**?

5. Consider the convolutional encoder in the figure below.



(a) Draw the state transition diagram for the convolutional encoder.

(b) Draw a section of the trellis for the convolutional encoder.

(c) Assuming that the initial state of the convolutional encoder is 00, and assuming a channel bit sequence 1101010011 is received, calculate the message sequence optimum in the maximum likelihood sense.