On Channel Estimation and Capacity for Amplify and Forward Relay Networks

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Abstract—Relay networks have received considerable attention recently, especially when limited size and power resources impose constraints on the number of antennas within a wireless sensor network. In this paper, we design and analyze a training based linear mean square error (LMMSE) channel estimator for time division multiplex amplify-and-forward (AF) relay networks. For the purpose of performance comparison we consider three distinct cases; In the first scenario, each relay estimates its backward and forward channels, in the second scenario each relay knows its backward and forward channels perfectly and finally in the third scenario relays have no knowledge of channels. Finally, we find a lower bound for the capacity considering the effect of training and estimation error.

I. INTRODUCTION

Next generation wireless networks are demanding high data rate services to accommodate requests from various applications. In order to provide reliable communications, one needs to compensate for the effects of signal fading due to multipath propagation and strong shadowing. One way to address these issues is to transmit the signal through one or more relays [1]. Different relay strategies have been studied in the literature with most of the attention focused on amplify-and-forward (AF), and decode-and-forward (DF) strategies. Initially, most of the prior work in this area assumed perfect channel knowledge at the relays and the receivers. However, recent publications such as [2] estimate the overall channel between the source and destination using LMMSE estimator assuming one relay. In [3] both Least Square (LS) and LMMSE channel estimators are derived for AF by using space time coding, such that each relay encodes its received signal using linear dispersion (LD) code (linear precoding).

In this paper, we extend current work by considering a training based LMMSE estimator for K relays in an AF configuration assuming a time division multiplex system such that the channel from relays to destination and vice versa are reciprocal. We derive a lower bound for the capacity of this relay network considering the effect of channel estimation error as part of noise at the destination. To contrast the results, we consider three cases; In the first scenario, each relay estimates its backward and forward channels, in the second scenario each relay knows its backward and forward channels perfectly and finally in the third scenario relays do not know their backward and forward channels.

II. SYSTEM MODEL AND PROBLEM FORMULATION

We consider an AF relay network consisting of 1 transmit antenna at the source, 1 receive antenna at the destination, and K relays, each equipped with 1 antenna. We denote by $h_{ti}$ the channel between the source and relay $i$, backward channel, while $h_{it}$ is the channel between the relay $i$ and the destination, forward channel. We assume that the channels follow the block-fading law, where the channels are constant for some coherence interval $T_c$, which is measured in symbols, and after that they change to an independent value which hold for another interval $T_c$. We further assume that channel estimation and data transmission is to be done during the interval $T_c$. Also backward and forward channels are independent and Rayleigh flat fading distributed which are reciprocal. We derive a lower bound for the capacity of the channel from relays to destination and vice versa are independent and Rayleigh flat fading distributed which are

$$r_i = h_{si} s + v_{si}, \quad (1)$$

where $r_i = [r_{i1}, \ldots, r_{ik}]^T$ is the received signal and $v_{si} = [v_{s1i}, \ldots, v_{sKi}]^T$ is the zero mean additive white complex Gaussian noise at the relay $i$ with covariance matrix $R_{v_i} = \sigma^2_{v_i} I$. Also the transmitter has the total power $E[s^*s] = T_c P_s$, where $P_s$ is the average transmitting power of the source. In the second phase each relay, multiplies its received signal by a scalar coefficient $\beta_i$ which is constant during coherence time $T_c$ and sends it, on the same time slot, to the destination. The received signal at the destination can be expressed as

$$y = \sum_{i=1}^{K} \beta_i r_i h_{ti} + v_t, \quad (2)$$

where $v_t$ is $T_c \times 1$ zero mean additive white complex Gaussian noise at the destination with covariance matrix $R_{v_t} = \sigma^2_{v_t} I$ and also independent of $v_{si}$ for all $i$. By plugging (1) in (2) the received signal can be expressed as

$$y = \sum_{i=1}^{K} \beta_i h_{si} h_{ti}, s + \sum_{i=1}^{K} \beta_i h_{ti}, v_{si} + v_t = h_{tot} s + n, \quad (3)$$

where $h_{tot}$ is the overall channel from the source to the destination and $n$ is the overall noise at the destination which is zero mean and has the covariance matrix

$$R_n = (\sigma^2_{v_t} \sigma^2_{h} \sum_{i=1}^{K} |\beta_i|^2 + \sigma^2_{v_i}) I = \sigma^2_{n} I. \quad (4)$$
Since the receiver does not know $h_{\text{tot}}$, training-based schemes assign part of the transmitted signal $s$ to be a known training signal from which the receiver can learn $h_{\text{tot}}$. Training based schemes are composed of two phases.

1) Training Phase: In this phase the received signal may be expressed as
\[ y_r = h_{\text{tot}} s_r + n_r \] (5)
where $s_r$ is the $T_r \times 1$ vector of training symbols sent over $T_r$ time symbols, and $y_r$ is the received vector at the receiver. We assume that $\sqrt{P_r}$ is sent as training symbols, then
\[ s_r = [\sqrt{P_r}, \ldots, \sqrt{P_r}]^T. \] (6)

2) Data Phase: In this phase the received signal may be expressed as
\[ y_d = h_{\text{tot}} s_d + n_d \] (7)
where $s_d$ is the $T_d \times 1$ vector of transmitted data symbols sent over $T_d$ time symbols.

It should be mentioned that both noise in training phase and data phase are zero mean and has the same covariance matrix as in (4), and also the average transmit power during training and data phase is the same which is $P_s$. While it is shown in [4] that in order to have signals received at the receiver co-phased, maximum SNR, each relay needs to compensate for the phase effect of its backward and forward channels, here we investigate three different scenarios and investigate the effect of relay functionality on the overall channel estimation.

A. Relays estimate backward and forward channels

In this scenario each relay needs to estimate its backward, $h_{s_i}$, and forward, $h_{t_i}$, channels by using training symbols. Since we are assuming that the relay network is a time division multiplex system, the channel between relays and destination, $h_{s_i}$, are reciprocal. In order to estimate the forward channel, after the transmitter sends control signals to the destination, the transmission starts from the receiver side by sending some training symbols to the relays, such that relays can estimate their forward channel. After that the transmitter starts sending training symbols to the destination through relays. The backward channel can be estimated at each relay by the same training symbols which is sent to the destination. Therefore, the transmission starts by sending training symbols from the transmitter to the destination through relays and there is no need for training symbols from the receiver and also estimation at each relay.

B. Relays know backward and forward channels perfectly

In this scenario, each relay knows its backward and forward channels perfectly and there is no need for training symbols from the receiver side. Here the transmission starts by sending training symbols from the transmitter, and relays just scale and forward the training symbols to the destination. The scaling factor is given by
\[ \beta_i = \sqrt{\frac{P_r}{\sigma_h^2 P_s + \sigma_v^2}} e^{-j[h_{s_i} + h_{t_i}]}, \] (10)
and
\[ T_c = T_r + T_d. \] (11)

C. Relays do not have knowledge of channels

In this scenario, relays just scale and forward its received signal to the destination and they do not have any knowledge of the backward and forward channels. Here, as in II-B, the transmission starts by sending training symbols from the transmitter to the destination through relays and there is no need for training symbols from the receiver and also estimation at each relay. Therefore,
\[ \beta_i = \sqrt{\frac{P_r}{\sigma_h^2 P_s + \sigma_v^2}}, \] (12)
and then we find a lower bound for the capacity considering the effect of training and estimation error.

III. CHANNEL ESTIMATION

We consider LMMSE estimator design to estimate the overall channel between the source and destination, and show its performance for the three scenarios mentioned above. First we start with the scenario in II-A and then show that the other two scenarios are special cases of this scenario. In order to do that we need to design channel estimator for the backward and forward channels at each relay and then derive channel estimator for the overall channel, $h_{\text{tot}}$. Since the channel estimator for $h_{s_i}$ and $h_{t_i}$ are the same, we consider designing channel estimator for the backward channel, $h_{s_i}$.

From (1) the received signal at each relay during training can be expressed as $r_i = h_{s_i} s_r + v_{s_i}$, where $s_r$ is defined in (6). Then the LMMSE estimation of $h_{s_i}$ can be expressed as
\[ \hat{h}_{s_i} = R_h s_r, R_r^{-1} r_i \] (13)
where $R_h s_r$ is the cross-covariance matrix between $h_{s_i}$ and $r_i$, also $R_r^{-1} r_i$ is the auto-correlation matrix of $r_i$, i.e.,
\[ R_h s_r = \mathbb{E}[h_{s_i} s_r^*] = \sigma_h^2 s_r^*, \] (14)
and
\[ R_r^{-1} r_i = \mathbb{E}[r_i r_i^*] = \sigma_v^2 I. \] (15)

Then the estimated channel can be expressed as
\[ \hat{h}_{s_i} = \frac{\sigma_v^2 \sqrt{P_s}}{T_r \sigma_h^2 P_s + \sigma_v^2} \sum_{j=1}^{T_r} r_{ij} = \frac{\sigma_h^2 \sqrt{P_s}}{T_r \sigma_h^2 P_s + \sigma_v^2} \left[ T_r \sqrt{P_s} h_{s_i} + v_{s_i,\text{tot}} \right]. \] (16)
where \( v_{x_{\text{tot}}} \sim \mathcal{CN}(0, T_r \sigma_v^2) \). In the same vein we can show that \( h_{s_i} \) has the same form as above.

Now we get back to estimating the overall channel, \( h_{\text{tot}} \), at the receiver. But before that we need to find some statistics of this overall channel. First we start by looking at the mean of \( h_{\text{tot}} \),

\[
\hat{h}_{\text{tot}} = \mathbb{E}[h_{\text{tot}}] = \mathbb{E} \left[ \sum_{i=1}^{K} \beta_i h_{s_i} h_{t_i} \right] = \sqrt{\frac{P_r}{\sigma_h^2 P_s + \sigma_v^2}} \sum_{i=1}^{K} \mathbb{E} \left[ h_{s_i} e^{-j \phi} h_{t_i} \right] \mathbb{E} \left[ h_{t_i} e^{-j \phi} h_{t_i} \right].
\]

(17)

Since \( h_{t_i} \) and \( h_{s_i} \) have the same statistical distribution we just need to find

\[
\alpha = \mathbb{E} \left[ h_{s_i} e^{-j \phi} h_{t_i} \right] = \mathbb{E} \left[ h_{s_i} e^{-j \phi} \right] \mathbb{E} \left[ h_{t_i} \right] = \mathbb{E} \left[ h_{s_i} e^{-j \phi} h_{t_i} \right].
\]

(18)

where

\[
\mathbb{E} \left[ h_{s_i} e^{-j \phi} h_{t_i} \right] = h_{s_i} \mathbb{E} \left[ e^{-j \phi} h_{t_i} \right].
\]

(19)

From (16) we can express \( \hat{h}_{s_i} \), as

\[
\hat{h}_{s_i} = \zeta_1 h_{s_i} + \zeta_2 v_{x_{\text{tot}}} = \zeta_1 |h_{s_i}| e^{j \angle h_{s_i}} + \zeta_2 v_{x_{\text{tot}}}
\]

(20)

where \( \zeta_1 = \frac{T_r \sigma_h^2 P_s}{\sigma_h^2 P_s + \sigma_v^2} \) and \( \zeta_2 = \frac{\sigma_v^2}{\sigma_h^2 P_s + \sigma_v^2} \). By rearranging (20) we can express it as

\[
|\hat{h}_{s_i}| = \frac{\zeta_1}{\zeta_1 |h_{s_i}|} e^{-j \angle h_{s_i}} + \frac{\zeta_2}{\zeta_1} e^{-j \angle h_{s_i}} v_{x_{\text{tot}}}.
\]

(21)

From (21) it can be seen that

\[
\mathcal{CN} \left( 1, \left( \frac{\zeta_2}{\zeta_1 |h_{s_i}|} \right)^2 T_r \sigma_v^2 \right),
\]

then

\[
\phi \triangleq \angle h_{s_i} - \angle h_{s_i} = \angle \mathcal{CN} \left( 1, \left( \frac{\zeta_2}{\zeta_1 |h_{s_i}|} \right)^2 T_r \sigma_v^2 \right).
\]

(22)

Now we can rewrite (19) as

\[
\mathbb{E} \left[ h_{s_i} e^{-j \phi} h_{t_i} \right] = h_{s_i} s_{\text{tot}} \mathbb{E} \left[ e^{-j \phi} \right] = |h_{s_i}| \mathbb{E} [\cos \phi] |h_{s_i}| - j |h_{s_i}| \mathbb{E} [\sin \phi] |h_{s_i}|.
\]

(23)

Finally by inserting (23) into (18) we can derive

\[
\alpha = \mathbb{E} \left[ h_{s_i} e^{-j \phi} h_{t_i} \right] = \mathbb{E} [|h_{s_i}| \mathbb{E} [\cos \phi] |h_{s_i}|] - j \mathbb{E} [|h_{s_i}| \mathbb{E} [\sin \phi] |h_{s_i}|] = \mathbb{E} [|h_{s_i}| \cos \phi] = \mathbb{E} [\cos \phi]
\]

(24)

which can be found numerically and is the same for all backward and forward channels since they have the same distribution. Then from (17) the overall channel mean can be expressed as

\[
\hat{h}_{\text{tot}} = \mathbb{E}[h_{\text{tot}}] = \sqrt{\frac{P_r}{\sigma_h^2 P_s + \sigma_v^2}} K \alpha^2.
\]

(25)

Now the mean of the received signal at the destination can be expressed as

\[
\bar{y}_r = \mathbb{E}[y_r] = \mathbb{E}[h_{\text{tot}} s_r + n] = s_r \mathbb{E}[h_{\text{tot}}],
\]

(26)

where \( s_r \) is defined in (6). Since \( h_{\text{tot}} \) and the received signal are not zero mean, the unbiased LMMSE estimation is given by

\[
\hat{h}_{\text{tot}} = \check{h}_{\text{tot}} + R_{\text{tot}y_r} R_{y_r y_r}^{-1} [y_r - \bar{y}_r],
\]

(27)

where \( R_{\text{tot}y_r} \) and \( R_{y_r y_r} \) are cross-covariance and covariance matrices of zero mean random variables \([h_{\text{tot}} - \hat{h}_{\text{tot}}, y_r - \bar{y}_r]\) i.e.,

\[
R_{\text{tot}y_r} = \mathbb{E}[(h_{\text{tot}} - \hat{h}_{\text{tot}})(y_r - \bar{y}_r)^*] = (\mathbb{E}[|h_{\text{tot}}|^2] - |\hat{h}_{\text{tot}}|^2) s_r^* = \sigma_h^2 s_r^*
\]

(28)

and

\[
R_{y_r y_r} = \mathbb{E}[(y_r - \bar{y}_r)(y_r - \bar{y}_r)^*] = \sigma_r^2 s_r^* s_r^* + \sigma_n^2 I,
\]

(29)

where \( \sigma_r^2 \) is the variance of the overall channel \( h_{\text{tot}} \). In order to find \( \sigma_h^2 \), as we already derived \( \hat{h}_{\text{tot}} \) in (25), we just need to find \( \mathbb{E}[|\bar{h}_{\text{tot}}|^2] \) which is

\[
\mathbb{E}[|\bar{h}_{\text{tot}}|^2] = \mathbb{E} \left[ \sum_{i=1}^{K} |\beta_i|^2 |h_{s_i}|^2 |h_{t_i}|^2 \right] + \mathbb{E} \left[ \sum_{i=1}^{K} \sum_{j=1, j \neq i}^{K} (\beta_i h_{s_i} h_{t_i})(\beta_j h_{s_j} h_{t_j}) \right] = \frac{P_r}{\sigma_h^2 P_s + \sigma_v^2} |K \sigma_h^4 + K (K - 1) \alpha^4|,
\]

(30)

where \( \alpha \) is defined in (24). Then the overall variance can be expressed as

\[
\sigma_{\text{tot}} = \mathbb{E}[|h_{\text{tot}}|^2] - |\hat{h}_{\text{tot}}|^2 = K \frac{P_r}{\sigma_h^2 P_s + \sigma_v^2} \left[ \sigma_h^4 - \alpha^4 \right].
\]

(31)

Now by inserting (25), (28), (29), and (31) into (27) the overall channel estimation will be

\[
\hat{h}_{\text{tot}} = \hat{h}_{\text{tot}} + \frac{\sigma_{\text{tot}}}{T_r P_s \sigma_h^2 + \sigma_v^2} s_r^* [y_r - \bar{y}_r],
\]

\[
= \hat{h}_{\text{tot}} + \gamma \left( T_r \sqrt{P_s} |\hat{h}_{\text{tot}} - \hat{h}_{\text{tot}}| + \sum_{i=1}^{T_r} n_i \right),
\]

(32)

where we defined \( \gamma \triangleq \frac{\sqrt{T_r \sigma_r^2}}{T_r P_s \sigma_h^2 + \sigma_v^2} \). As it could be seen, the channel estimator is unbiased i.e., \( \mathbb{E}[\hat{h}_{\text{tot}}] = \mathbb{E}[h_{\text{tot}}] = \hat{h}_{\text{tot}} \) and has the variance

\[
\sigma_{\text{tot}}^2 = \mathbb{E}[|h_{\text{tot}} - \hat{h}_{\text{tot}}|^2] = \gamma^2 \left[ T_r^2 P_s \sigma_h^4 + T_r \sigma_v^2 \right].
\]

(33)

We can break the overall channel \( h_{\text{tot}} \) into

\[
\hat{h}_{\text{tot}} = \hat{h}_{\text{tot}} + \hat{h}_{\text{tot}},
\]

(34)

where \( \hat{h}_{\text{tot}} \) is the channel estimation error which is zero mean i.e., \( \mathbb{E}[\hat{h}_{\text{tot}}] = 0 \). Also since the estimator is LMMSE, \( h_{\text{tot}} \) and \( \hat{h}_{\text{tot}} \) are orthogonal [5], then

\[
\sigma_{\text{tot}}^2 = \sigma_h^2 + \sigma_{\text{tot}}^2
\]

(35)
and by inserting (33) into (35) the estimation error variance can be expressed as
\[ \sigma_{\hat{h}_{tot}}^2 = (1 - \gamma^2 T^2 P_s) \sigma_{\hat{h}_{tot}}^2 - \gamma^2 T \sigma_n^2 . \] (36)

For the other two scenarios we just need to modify \( \alpha \) in (24) and all the rest of solutions are intact. In scenario II-B, where the relays know their backward and forward channels perfectly, \( \alpha = \mathbb{E}[h_{s_i} e^{-j \theta_{s_i}}] = \mathbb{E}[|h_{s_i}|] = \sqrt{\frac{\sigma_x^2}{4}} \) and for the scenario II-C, where relays have no knowledge of channels, \( \alpha = \mathbb{E}[h_{s_i}] = 0 \).

IV. TRAINING BASED CAPACITY BOUND

The capacity in bits per channel use can be written as
\[ C = \sup_{p(s_d), E[|s_d|^2]=T_d} \frac{1}{2} \frac{1}{T_c} I(y_d; s_r, y_r; s_d) , \] (37)
where the capacity is scaled by a factor of 2 due to the two-phase protocol scheme. Now
\[ I(y_d; s_r, y_r; s_d) = I(y_d; s_d|y_r, s_r) + I(y_r, s_r; s_d) \]
(38)
where \( I(y_r, s_r; s_d) = 0 \) because \( s_d \) is independent of \( y_r \) and \( s_r \). During the data phase the received signal can be express as
\[ y_d = \hat{h}_{tot} s_d + \hat{n}_{tot} \]
(39)
where \( \hat{h}_{tot} \) is the LMMSE channel estimate defined in (32) and \( \hat{h}_{tot} = h_{tot} - \hat{h}_{tot} \) is the zero mean estimation error defined in (34) and (35). Since \( \hat{h}_{tot} \) is known to the receiver, the capacity of a training based relay system is equivalent to a known channel system subject to additive noise with power
\[ \sigma_{n_{tot}}^2 = \frac{1}{T_d} \mathbb{E}[|n_{tot}^*|^2] = \frac{1}{T_d} \mathbb{E}[|\hat{h}_{tot} \hat{h}_{tot}|] \mathbb{E}[|s_d^* s_d|] + \frac{1}{T_d} \mathbb{E}[|n_d^* n_d|] = \frac{1}{T_d} \sigma_{\hat{h}_{tot}}^2 T_d P_s + \frac{1}{T_d} T_d \sigma_n^2 = \sigma_{\hat{h}_{tot}}^2 T_d P_s + \sigma_n^2 \]
(40)
It should be mentioned that the additive noise \( n_{tot} \) in (39) is not gaussian and independent of data. In order to find the capacity of known channel relay system we need to find the effect the worst effect the additive noise can have on the capacity which leads us to the lower bound
\[ C \geq C_{\text{worst}} = \inf_{p(n_{tot}), E[|n_{tot}|^2]=T_d \sigma_{n_{tot}}^2} \sup_{p(s_d), E[|s_d|^2]=T_d P_s} I(y_d; s_d|\hat{h}_{tot}) . \] (41)

It is shown in [6], Theorem 1, that if the signal and additive noise are uncorrelated, the worst case noise has zero-mean gaussian distribution with the same power of the additive noise. Here the additive noise, \( n_{tot} \), and signal \( s_d \) are uncorrelated since we use LMMSE estimator because
\[ \mathbb{E}[n_{tot}^* s_d | y_r, s_r] = \mathbb{E}[\hat{n}_{tot}^* s_d | y_r, s_r] + \mathbb{E}[n_d^* s_d | y_r, s_r] = \mathbb{E}[\hat{n}_{tot}^* | y_r, s_r] \mathbb{E}[s_d^* s_d | y_r, s_r] + 0 = 0 \]
(42)
and \( \mathbb{E}[\hat{n}_{tot}^* | y_r, s_r] = \mathbb{E}[\hat{n}_{tot}^* - \hat{h}_{tot}^* | y_r, s_r, s_r] = 0 \). Then we may write
\[ C \geq \mathbb{E} \left[ \frac{1}{2} \frac{T_d}{T_c} \log \left( 1 + \frac{P_s \sigma_{\hat{h}_{tot}}^2}{P_s \sigma_{\hat{h}_{tot}}^2 + \sigma_n^2} \right) \right] . \] (43)

V. SIMULATION RESULTS

In this section, first we investigate the performance of our proposed channel estimation and then we study its effect on capacity lower bound. We define the signal to noise ratio as \( \text{SNR} = \frac{P_s}{\sigma_n^2} \), where \( \sigma_n^2 = \sigma_{\hat{s}}^2 \) and also \( \sigma_{\hat{s}}^2 = 1 \). We further assume that the transmitter output power, \( P_s \), and the relays average output power, \( P_r \), are set to 10 dB.

Figure 1 shows channel estimation mean square error (MSE) for different scenarios versus SNR for coherence interval of \( T_c = 300 \) and training interval of \( T_T = 10 \). It can be seen that increasing the number of relays increases the MSE which is consistent with (36) where it says that estimation error variance increases with \( K \). It also compares MSE channel estimation of different scenarios. It can be seen that the estimator performs the same for different schemes except at low SNR which by having knowledge of channels we can get around 0.5 dB gain. Figure 2 shows capacity versus coherence time \( T_c \) for the first scenario, where each relay estimates its backward and forward channels, with SNR=10 dB and \( K=10 \) for \( T_T = 10, 20 \). It can be seen that for \( T_T = 10 \) the capacity is higher compared to when \( T_T = 20 \) especially at low \( T_T \) and as \( T_T \) increases they get closer. This happens despite the fact that by increasing \( T_T \) we will have better channel estimation and consequently better capacity, but here the dominant factor is \( \frac{T_d}{T_c} = T_c T_T \). In order to make the point more clear, Fig. 3 shows the capacity versus \( T_T \) for coherence interval of \( T_c = 300 \) and SNR=10 dB. It can be seen that the maximum capacity happens at \( T_T = 5 \) and then decreases very fast. The reason is that the decrease in capacity is linear with respect to \( \frac{T_d}{T_c} = \frac{T_T}{T_c} \) while the increase in capacity is logarithmic with respect to increase in effective signal to noise ratio, \( \text{SNR}_{\text{eff}} \). Figure 3 also shows that when the receiver knows the overall channel perfectly, for small \( T_T \) capacity lower compared to bigger values of \( T_T \). The reason is that for small \( T_T \) the relays do not have a good estimate of backward and forward channels and increasing \( T_T \) would result in better estimation at relays and finally signals get to the destination co-phase which is the reason that the capacity becomes constant. Similar results, like Fig. 2 and 3, can be
drawn for the other two scenarios, not shown here, except that the capacities will be lower. Figure 4 shows capacity versus SNR for the three scenarios where $T_c = 300$, $T_r = 10$, and $K = 10$. As it is expected knowledge of channels at the relays improves the capacity, which is due to the fact that having a better knowledge of channels at the relays will result in adding signals at the destination constructively.

VI. Conclusions

In this paper we proposed a training based channel estimation for AF relay networks with $K$ relays for three different scenarios. We considered LMMSE channel estimator and showed that the case where the relays estimate their immediate channels is the general case and the channel estimation at the receiver for the other scenarios can be found easily. We also derived a lower bound for the capacity considering the effect of channel estimation error. Simulations are provided to validate our studies.

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