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Dear Colleague:

On behalf of the Technical Program Committee, we are pleased to inform you that your paper has been accepted for presentation at the 1997 IEEE AP-S International Symposium and URSI North American Radio Science Meeting. The response to the Calls for Papers was overwhelming. Approximately 1300 papers were received which necessitated scheduling five full days of sessions from Monday through Friday.

Title: Time Domain Vector Potential Formulation for the Solution of Electromagnetic Problems, F. DE FLAVIIS, M. NORO, R.E. DIAZ, N.G. ALEXOPOULOS, University of California at Los Angeles, CA, USA

Paper Number: 50.02

Date and Time: TUESDAY, July 15, 1997 at 13:30

Twenty minutes will be allocated for your presentation. It is very important that your paper and discussion not exceed 20 minutes because we are operating on a very tight schedule. An overhead projector will be provided in each lecture room, and a 35mm slide projector where this has been requested. No other equipment will be provided.

In multi-authored papers, you are the only person being contacted so it is important that you inform the other authors about this notice, and the time your paper is scheduled. In the event that you have to cancel your paper, please notify the undersigned as soon as possible.

Registration and accommodation information can be found at our WWW address: http://www.nrc.ca/conferv/apsursi97/welcome.html. The advance program will also be available on this site within a few weeks. Please note that the deadline for registering without a late penalty is May 30, 1997.

Yours sincerely,

Doris Ruest (Mrs.)
Conference Manager
TIME DOMAIN VECTOR POTENTIAL FORMULATION FOR THE SOLUTION OF ELECTROMAGNETIC PROBLEMS

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ABSTRACT
Several techniques have been proposed for the solution of Maxwell’s equations, such as FDTD, which rely on discretization of Maxwell’s equations in the time. These techniques are attractive because of their simplicity but are limited to dealing with structures with low dispersion characteristics. Other techniques as condensed TLM offer superior characteristics in terms of dispersion but are more demanding in terms of computer resources. Attempts to use the vector potential formulation by discretization of the vector potential wave equation have also been made in the past. Although the scheme is attractive because of some of the advantages of the TLM technique, they have the shortcoming of the difficulty of implementing metal boundaries. In this paper a new technique based on discretization of Maxwell’s equations in the vector potential form (VP) is presented. This new technique maintains the advantage of condensed node representation as in the vector potential formulation, but offers an easy way to treat metal boundaries.

1. VECTOR POTENTIAL FORMULATION
Upon introduction of the vector potential [1] it is possible to recast Maxwell’s equations in the following form[2]:

\[
\begin{cases}
\mu \frac{\partial (\varepsilon \mathbf{E})}{\partial t} = \nabla (\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A} - \nabla \mu \times \mathbf{H} - \sigma \mu \mathbf{E} \\
\frac{\partial \mathbf{A}}{\partial t} = -\mathbf{E} - \nabla \phi \\
\nabla \cdot \mathbf{A} = -\varepsilon \mu \frac{\partial \phi}{\partial t}
\end{cases}
\]  

were \( \mathbf{A} \) and \( \phi \) are respectively the electric vector potential and the scalar potential. If we restrict our attention to the two dimensional case, all the space derivatives in the z-direction disappear, so we obtain
\[
\begin{align*}
\frac{\partial E_z}{\partial t} & = -\frac{1}{\varepsilon} \left[ \frac{\partial}{\partial x} \left( \frac{1}{\mu} \frac{\partial A_z}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{1}{\mu} \frac{\partial A_z}{\partial y} \right) \right] - \frac{\sigma}{\varepsilon} E_z \\
\frac{\partial A_z}{\partial t} & = -E_z
\end{align*}
\]  
(2)

where it is clear that in the lossless case \( \sigma \) is equal to zero. The 3-D case can be treated as well, but it is are omitted here for sake of simplicity.

2. SCATTERING PROBLEM

As a validation of the possibility of treating lossy dielectric materials, we present in this section results of simulation for the internal electric field of a uniform, circular dielectric cylindrical scatterer. The cylinder is assumed to be infinite in the z-direction. The incident radiation is assumed to be a TM wave with respect the axis of symmetry of the cylinder. Because there is no variation of either scatterer geometry or incident field in the z-direction, this problem may be treated as the 2-dimensional scattering of the incident wave. The grid coordinates internal to the cylinder with radius 0.06m are assigned the dielectric parameters. All the grid points outside this grid are assigned the parameters of free space. The program is time-stepped for a long enough time so that the plane wave is scattered from the cylinder and the scattered field reaches the observation region. DFT algorithm is used to extract the information of the field distribution of the frequency of 1.5 GHz. The exact solution, is calculated using the summed series technique as in Jones[3].

The computer solution locates the positions of all the peaks and nulls of the envelope of the electric field with error less then 0.3%. For the first example the cylindrical scatterer has the same parameters, except that it is lossy with a dielectric constant \( \varepsilon_d = 2.0 \) and conductivity \( \sigma_e = 0.0356 \) S. The result of this simulation is reported in Fig.1. An analogous simulation was run with the same
geometrical parameters and the same pulse shape, but for a magnetic cylinder ($\mu_r = 2.0$). The result is shown in Fig. 1, and the comparison is made with Finite Difference Time Domain (FDTD) [4,5] calculations for the same case. In all the considered cases computation times and memory requirement were identical using FDTD and VP.

3. FREQUENCY DEPENDENT MATERIALS

In the case of an electric Debye material, we proceed from eq. (2), which holds in the one dimensional case. We can imagine the total polarization $P_z$ as the sum of two polarizations $P_z^{(1)}$ and $P_z^{(2)}$, so that the "effective" average polarization is $P_z = P_z^{(1)} + P_z^{(2)}$, where it is clear that each polarization can be though as to act independently from the other. Each polarization obeys the differential equation

$$P_z^{(i)} + \tau_i \frac{\partial P_z^{(i)}}{\partial t} = \varepsilon_p g_i E_z,$$

where $\tau$ is the time constant, $\varepsilon_p$ the difference between $\varepsilon_\infty$ and $\varepsilon_s$, and $g_i$ the weight factor of each pole. Substituting eq. (3) into (2), we obtain

$$\frac{1}{\varepsilon_p} \frac{\partial P_z}{\partial t} = -g_1 \left( \frac{P_z^{(1)}}{\varepsilon_p g_1} - E_z \right) - g_2 \left( \frac{P_z^{(2)}}{\varepsilon_p g_2} - E_z \right)$$

$$\frac{\partial E_z}{\partial t} = -\frac{1}{\varepsilon} \left[ \frac{\partial}{\partial x} \left( \frac{1}{\mu} \frac{\partial A_z}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{1}{\mu} \frac{\partial A_z}{\partial y} \right) \right] - \frac{1}{\varepsilon} \frac{\partial P_z}{\partial t}$$

4. BACK-SCATTERING EXPERIMENTS

Let us consider a plane wave incident upon the flat infinite air-medium interface; because of the simple geometry the problem can be reduced to one dimension [1]. The one dimensional space consists of 1000 cells: 700 are used to model the free space (air) and the remaining 300 are used for the complex material. Each cell corresponds to a length of 0.1 mm and the time step is 0.25 psec. The incident wave is a Gaussian pulse with maximum frequency of 200 GHz and width of 20 time steps. The simulation is time stepped for a long enough time until the pulse reaches the interface and is partially reflected. The reflection coefficient as a function of the frequency is therefore calculated as the ratio of the spectrum of the reflected and the incident wave. The calculated reflection coefficient is compared to the correspondent analytical quantity obtained in the frequency domain from $R(\omega) = \frac{\eta_0 - \eta}{\eta_0 + \eta}$, where $\eta_0$ and $\eta_1$ are the characteristic impedance of free space and the complex medium respectively, $\eta_0 = \sqrt{\mu_0 / \varepsilon_0}$ and $\eta_1 = \sqrt{\mu_1(\omega) / \varepsilon_1(\omega)}$. In the
first experiment we consider the water-air interface; the complex permittivity of water can be approximated by a single order Debye relaxation. We have used $\varepsilon_s = 81.0$, $\varepsilon_\infty = 1.8$, and $\tau_0 = 9.4 \times 10^{-12}$ sec[6]. In the second experiment we studied the reflection coefficient at the interface between air and a two pole electric Debye material, for which we chose the following values: $\varepsilon_s = 100.0$, $\varepsilon_\infty = 4.0$, $\tau_1 = 10^{-11}$ sec, $\tau_2 = 5.3 \times 10^{-11}$ sec, $g_1 = 0.7$, $g_2 = 0.3$. The poles have been chosen in such a way that the two relaxation times are well separated. The results for the reflection coefficients are plotted in Fig. 2.

![Graph](attachment:image.png)

**Fig.2** Reflection coefficient for air-water interface

5. CONCLUSIONS

A new approach for the solution of scattering problems has been proposed. The scheme takes advantage of the simplicity of the leap-frog scheme between the electric field and the vector potential, but maintains the higher performances of TLM condensed node due the nature of the discretization.

REFERENCES